Offshoring and Inflation*

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Abstract

Did trade integration suppress inflation in the United States? We say no, in contradiction to the conventional wisdom. Our answer leverages two basic facts about the rise of trade: offshoring accounts for a large share of it, and it was a long-lasting, phased-in shock. Incorporating these features into a New Keynesian model, we show trade integration was inflationary. This result continues to hold when we extend the model to account for US trade deficits, the pro-competitive effects of trade on domestic markups, and cross-sector heterogeneity in trade integration in a multisector model. Further, using the multisector model, we demonstrate that neither cross-sector evidence on trade and prices, nor aggregate time series price level decompositions are informative about the impact of trade on inflation.

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Recent decades have seen large increases in trade integration. On the flip side, recent trade wars and other de-globalizing forces have policymakers concerned about the macroeconomic fallout of unwinding international integration. In this paper, we shed light on the macro-effects of trade (dis-)integration by looking backwards to the rise of trade since the mid-1990s in the United States. The particular question we ask is monetary in nature: has globalization in general, and trade integration in particular, suppressed inflation? Flipping this question around, will de-globalization let the inflation genie back out of the bottle?

To answer this question, we develop an open economy new Keynesian (NK) framework with trade in both intermediate inputs and final goods, and we apply the framework to analyze the inflationary impacts of rising trade in the United States during recent decades. The framework extends the canonical small open economy NK model [Galí and Monacelli (2005); Galí (2015)] to incorporate "offshoring" – the use of foreign intermediate inputs in production – in addition to trade in final goods.[1] This extension is more than window dressing: increases in offshoring are large in the data, so any quantitative account of the impacts of trade on inflation must emphasize offshoring. Nonetheless, offshoring is omitted from workhorse NK models; Consequently, we will argue that its impacts are commonly misunderstood in policy discussions [Carney (2017, 2019)].

In addition to this extension, we also apply the model in new ways to analyze the rise of trade. First, we use domestic sourcing shares as "sufficient statistics" to assess the impacts of trade in the model, borrowing from the international trade literature [Costinot and Rodríguez-Clare (2014)].[2] This allows us to overcome challenges regarding measurement of the impacts of trade on prices in the data, and to sidestep thorny questions about currency invoicing. Further, it enables us to analyze the impacts of trade in a concise "three equation model," in which dynamics of the domestic sourcing shares influence inflation via both the Phillips Curve and IS curve. Organizing the model in this way also allows us to treat changes in sourcing shares as shocks in analysis of retrospective inflation dynamics.

Second, we analyze the impact of long-lived (arguably permanent) shocks to trade openness, with long phase-in dynamics. Allowing for permanent, phased-in shocks captures an essential feature of the data – globalization involved a shift in steady states, from a less open to more open world, which took place slowly over time. We show that taking this aspect of globalization into account is important for inflation dynamics in the open economy NK model. Permanent, phased-in shocks yield inflation dynamics that are completely different than the temporary shocks typically analyzed in linearized NK models.

To open the analysis, we develop an accounting framework to link changes in trade to output and consumer prices. This framework serves two purposes. First, it highlights that there are two channels via which trade may impact consumer prices. The "old" channel operates via trade in consumption goods directly. Falling prices for imported consumption goods, and substitution of imports for domestic goods, lowers the consumer price level. The "new" channel that we emphasize operates via the use of imported inputs in production of domestic goods. Falling prices for imported inputs reduce domestic production costs, and substitution from domestic to foreign input suppliers amplifies this decline. Lower production costs then bring down prices for domestically produced goods and services. Further, because industries are connected to one another via behind-the-border input linkages, exposure to this offshoring-driven decline in costs depend not only on an industry's own sourcing behavior, but also that of its upstream industry suppliers.[3]

[1] Other recent open economy monetary models that incorporate inputs include Gopinath et al. (2020) and Auray, Devereux and Eyquem (2020). Relatedly, Amiti, Itskhoki and Konings (2014) study the impact of imported inputs on pricing in a real model.

[2] Our approach is also related to the macro-application of sufficient statistics in Baqaee and Fahri (2019), though we apply in a monetary (rather than real) context.

[3] These direct and indirect input cost effects are qualitatively similar to the role of firm-level changes in input sourcing

We use this framework to perform an accounting exercise that links changes in trade to industry-level output prices, and then industry-level output prices together with import prices to aggregate consumer prices. Using data from the BEA, we show that the rise of offshoring drives industry-level price dynamics: industries that are more exposed to offshoring have experienced lower output price inflation over the long run. Further, increases in trade over time appear to restrain consumer price inflation, lowering consumer price inflation by 10-40 basis points per year. Increases in offshoring are as important as increases in foreign sourcing of consumption goods in this result. This demonstrates both the potential importance of trade, and the importance of offshoring in particular, in accounting for aggregate prices.

Motivated by these accounting results, we then turn to a full-fledged NK model. In this transition, we emphasize that data alone cannot answer important macro-counterfactual questions about inflation. At a very basic level, inflation is a monetary phenomenon – endogenous model responses to trade shocks combined with the conduct of monetary policy determine inflation.[4] Further, inflation is a forward-looking variable in the NK framework, so both contemporary and future changes in trade matter for determining current inflation. Thus, in addition to the facts developed via the accounting exercise, we need a model to provide a full analysis of inflation dynamics.

Using the model, we argue that the basic accounting results we presented at the outset are surprisingly uninformative about the role of rising trade in explaining inflation. Given the historical path of shocks, the NK model predicts that the rise of trade during the late 1990's and early 2000's was inflationary, rather than deflationary.

To unpack this result, we collapse the full model down to an equivalent "three equation model" – including the NK Phillips curve, the dynamic IS curve, and the (inflation targeting) monetary policy rule. We show that only shocks to domestic sourcing of final goods shift the Phillips Curve. Further, because changes in domestic sourcing of final goods were relatively small over this period, this channel is quantitatively modest. In contrast, both offshoring and final goods trade shocks appear in the IS curve – specifically, the time path for these shocks is embedded in the real natural rate of interest. A sustained rise in offshoring – where domestic sourcing shares are falling over time – generates an increase in the real natural rate, which is itself inflationary. This dynamic response to the rise in offshoring over time drives current inflation up at the outset.

We extend the model in three main ways to probe this result. First, we incorporate financial inflow shocks into the model to match changes in the trade balance over time, which allow the model to match the dynamics of the global savings glut in the early 2000's. We find that capital inflow shocks have minor effects on inflation in the model; Surprisingly, the rise of the US trade deficit in the early 2000's actually pushes inflation up, though the magnitude of this effect is modest. Second, we introduce variable markups in the model, by that preferences and technologies take the Kimball (1995) form (as in Gopinath et al. (2020)). This set up allows trade to have pro-competitive effects, whereby increases in foreign sourcing of final goods and inputs restrains markups set by domestic producers. Despite these pro-competitive effects, we find again that trade integration increases inflation, even more than in the baseline model due to general equilibrium effects on the supply side of the economy by which lower markups raise domestic output.

analyzed by Blaum, Lelarge and Peters (2018). Our study of them is both more aggregated (sector-level rather than firm-level) and focused on the monetary implications of these shocks.

[4] Just to fix ideas clearly, suppose that the central bank has a price level (not inflation targeting) objective. In this event, the central bank would perfectly offset any impacts of trade shocks on inflation. This thought experiment emphasizes that how the economy and monetary policy respond to trade shocks matters for interpreting the accounting results. Nonetheless, the actual explanation we give for how inflation responds to trade shocks is not highly contingent on the specifics of policy, as long as the central bank does not perfectly target the price level.

Third, we build out the model to include multiple sectors, with heterogeneous sourcing dynamics across sectors and end uses. In the multisector model, we again find trade integration is inflationary. Further, we show the multisector model yields a decline in the relative price of manufacturing output, as observed in the data. Further, rising trade appears to restrain trade – in an accounting sense – in the model, as in our initial exploration of the data. These results serve to emphasize that one cannot draw conclusions about the impact of trade on inflation from studies linking cross-sectional changes in output prices to changes in trade, or studies that decompose prices into components attributable to domestic and import prices.

Given our conclusion, it is worth pointing out that academics and policymakers almost universally think that globalization was deflationary. For example, Carney (2019) states: "The integration of low-cost producers into the global economy has imparted a steady disinflationary bias."[5] This follows a large literature that has studied the role of trade in explaining the slope of the Phillips Curve, the role of "global slack" in inflation dynamics, and inflation synchronization across countries [Romer (1993); Rogoff (2003); Ball (2006); Rogoff (2007); Bianchi and Civelli (2015); Carney (2017); Auer, Levchenko and Saurè (2019); Forbes (2019)].[6] Our conclusions differ from prevailing consensus because our analysis captures two salient features of globalization overlooked in prior work. First, existing approaches largely ignore input trade, and input trade impacts inflation differently than does trade in final goods. Further, in our empirical context, changes in input trade are of first order importance. Second, existing work studies short-run dynamics for transitory shocks, while we study medium-term dynamics for persistent (possibly permanent) shocks.

Our paper is also related to a body of work that shows that higher import penetration leads to lower price inflation at the industry level. Using differences-in-differences style empirical designs, recent papers have documented this result for both consumer prices [Bai and Stumpner (2019); Jaravel and Sager (2019)] and output prices [Auer and Fischer (2010); Auer, Degen and Fischer (2013)]. One clarifying point to make is that these papers focus on the role of imports in providing competition for domestic producers, which restrains markups on domestic goods. In contrast, our baseline model emphasizes the imported input cost channel, which is operative even with constant markups. We do consider variable markups in an extension to this baseline model, however. Beyond this, we also emphasize that these differences-in-differences type results – though informative about particular mechanisms operative in the data – do not directly translate to results for inflation, which is a general equilibrium, monetary phenomenon.

Lastly, our work is also broadly informed by recent work on trade dynamics. We adopt a perfect foresight approach in analyzing trade shocks in the model, similar to Eaton et al. (2011), Reyes-Heroles (2016), Kehoe, Ruhl and Steinberg (2018), and Ravikumar, Santacreu and Sposi (2019).[7] Whereas these papers focus on real outcomes following trade shocks, we study a monetary economy. The framework in our paper can also be applied to analysis of the inflationary impacts of trade policy, which connects to recent papers on trade policy and macro outcomes [Erceg, Prestipino and Raffo (2018); Barbiero et al. (2018); Barattieri, Cacciatore and Ghironi (2019)]. Finally, in analyzing changes in trade in a model with nominal rigidities, our paper is also related to Rodríguez-Clare, Ulate and Vasquez (2020), who examine the impacts of the China shock on

[5]Carney is far from alone in this opinion. Yellen (2006) states: "the IMF calculates that non-oil import price reductions lowered US inflation by an average of 1/2 percentage point a year over 1997 to 2005. These results are in line with those from...the Federal Reserve Board that estimates that lower (core) import prices have reduced core US inflation by an annual average of 1/2 to 1 percentage point over the last 10 years." See also IMF (2006) and Bean (2007).

[6]Much of this literature focuses on how globalization changes the slope of the Phillips Curve. It is therefore useful to point out that our argument is entirely different. In fact, because we solve the model via linearization around an initial equilibrium prior to the rise of trade, the slope of the Phillips Curve is held fixed in our analysis. We could, however, also solve the model exactly (without linearization), in which case the slope of the implicit Phillips Curve would be changing over time. We have in fact done this, and it does not substantially change our argument, thus we prefer to focus on the simpler linearized model.

[7]Relatedly, Alessandria and Choi (2019) study very persistent, but stationary trade cost shocks.

employment and unemployment across local labor markets in a model with nominal wage rigidity.

1 An Account of Offshoring, Trade, and Inflation

In this section, we develop an accounting framework that ties the rise of trade to industry-level and aggregate prices. This framework shares its basic structure with the multisector model that we analyze in Section 4. However, we present the framework here with a few simplifying assumptions, in order to get to data as quickly as possible. We then revisit this accounting exercise in the complete multisector model in Section 4.

To begin, we characterize how industry-level prices for domestic output depend on offshoring – the use of imported inputs, both in the producer's own sector and in upstream sectors – and domestic factor costs. We then discuss how the aggregate consumer price level is linked to changes in the prices of domestic output versus imported final goods. Combining these results, we decompose changes in the aggregate consumer price level into components attributable to changes in offshoring, domestic factor costs, and imported consumption goods. We implement this decomposition using input-output and price data for the United States from 1997-2018.

1.1 Prices for Domestic Output

Consider a two country environment, with Home (H) and Foreign (F) countries, and many industries $s \in \{1, \ldots, S\}$.[8] Within each Home industry, there is a unit continuum of varieties, which are produced under monopolistic competition. These varieties are aggregated into composite goods, which are then used at Home as final or intermediate goods and exported. Each producer has a nested, constant elasticity of substitution (CES) production function. At the top level, they substitute between labor and a composite intermediate input, in a Cobb-Douglas production function.[9] At the middle level, they substitute across inputs originating from different upstream sectors in forming the composite input, again with Cobb-Douglas aggregation. At the bottom level, they substitute between sector-level inputs coming from Home versus Foreign sources, which are aggregated via a CES function. Because this basic CES monopolistic competition structure is standard, we jump right into discussion of results for prices that come from it.

Prices for each Home variety can be written as a time-varying markup over marginal costs, and all producers are symmetric. Thus, Home sector-level output prices in sector s at date t are given by:

$$P_{Ht}(s) = \mu_t(s) MC_t(s) \tag{1}$$

$$\text{with} \quad MC_t(s) = Z_t(s)^{-1} W_t^{1-\alpha(s)} P_{Mt}(s)^{\alpha(s)}, \tag{2}$$

where $\mu_t(s)$ and $MC_t(s)$ are time-varying markups and marginal costs in sector s. Marginal costs depend on productivity ($Z_t(s)$), the price of a composite primary factor (W_t), and the price of a sector-specific composite input ($P_{Mt}(s)$). The parameter $\alpha(s)$ is the Cobb-Douglas share for the composite input in total

[8] Our focus will be entirely on accounting for changes in Home output and consumer prices, so Foreign is merely a composite source of imports.

[9] We adopt Cobb-Douglas production functions here in order to simplify the exact solution for this model. We adopt a similar assumption for preferences below. Both assumptions are consistent with how we parameterize the multisector model for quantitative analysis in Section 4.

costs.[10] The composite input price in turn is given by:

$$P_{Mt}(s) = \prod_{s} P_t(s', s)^{\alpha(s',s)/\alpha(s)} \tag{3}$$

$$\text{with} \quad P_t(s^{'}, s) = \left[\xi(s', s) P_{Ht}(s')^{1-\eta(s')} + (1 - \xi(s', s)) \left(\tau_{Mt}(s') P_{Ft}(s')\right)^{1-\eta(s')}\right]^{1/(1-\eta(s'))}, \tag{4}$$

where $P_t(s', s)$ is the composite price of inputs purchased from sector s' by sector s, $\alpha(s', s)/\alpha(s)$ is the share of inputs from s' in total input expenditure by sector s, $P_{Ft}(s')$ is the price of a composite bundle of foreign varieties, and $\tau_{Mt}(s')$ is an iceberg-type trade cost paid on imports of intermediate inputs.

Consider now the change in prices over time interval $[0, t]$. Further, for a given variable X_t, let $\hat{X}_t = X_t/X_0$ denote the ratio of its value in period t to its value in the base period. Then, the price of output in period t relative to baseline is:

$$\hat{P}_{Ht}(s) = \hat{\mu}_t(s) \widehat{MC}_t(s) = \hat{P}_{Vt}(s)^{1-\alpha(s)} \hat{P}_{Mt}(s)^{\alpha(s)}, \tag{5}$$

where $\hat{P}_{Vt}(s) = \hat{W}_t \left(\hat{\mu}_t(s)/\hat{Z}_t(s)\right)^{1/(1-\alpha(s))}$ is the price of real value added in sector s (equivalently, the sector-specific GDP price deflator). Changes in composite input prices are:

$$\hat{P}_{Mt}(s) = \prod_{s} \hat{P}_t(s', s)^{\alpha(s',s)/\alpha(s)} \tag{6}$$

$$\hat{P}_t(s^{'}, s) = \left[\Lambda^M_{H0}(s', s) \hat{P}_{Ht}(s')^{1-\eta(s')} + \Lambda^M_{F0}(s', s) \hat{P}_{Ft}(s')^{1-\eta(s')}\right]^{1/(1-\eta(s'))}, \tag{7}$$

where $\Lambda^M_{it}(s', s) = \frac{P_{it}(s')M_{it}(s',s)}{P_t(s',s)M_t(s',s)}$ is the share of input spending on inputs from country $i \in \{H, F\}$ and sector $s'$by Home sector s in total spending in inputs from s' by s by Home, and $\Lambda^M_{i0}(s', s)$ is the base period value of these shares.

This exposition suggests that we could quantify the impact of offshoring on gross output prices using import price data. That is, we could measure price changes for foreign goods $\hat{P}_{Ft}(s')$, and feed them through Equations 5-7 to arrive at predicted changes in output prices. While this approach is straightforward in the model as written, it is not a practical route forward due to shortcomings in standard data sources; We discuss these shortcomings further in Appendix A.

Instead, we proceed here by invoking a sufficient-statistics argument. Using the usual first order conditions for the purchases of domestic inputs, we can write the expenditure share on domestic goods as: $\Lambda^M_{Ht}(s', s) = \frac{P_{Ht}(s')M_{Ht}(s',s)}{P_t(s',s)M_t(s',s)} = \xi(s', s) \left(\frac{P_{Ht}(s')}{P_t(s',s)}\right)^{1-\eta(s')}$. Taking ratios across time, we write $\hat{P}_t(s', s)$ as follows:

$$\hat{P}_t(s', s) = \hat{P}_{Ht}(s') \hat{\Lambda}^M_{Ht}(s', s)^{1/(\eta(s')-1)}. \tag{8}$$

This expression is an analog to the sufficient-statistics approach to counterfactual analysis of the the gains from trade, advocated by Arkolakis, Costinot and Rodríguez-Clare (2012) and Costinot and Rodríguez-Clare (2014). Like Blaum, Lelarge and Peters (2018), we apply it on the production side of the economy. When the share of inputs sourced domestically falls ($\hat{\Lambda}^M_{Ht}(s', s) < 1$) – i.e., when offshoring rises – then the cost of the input bundle falls, as long as foreign inputs are gross substitutes for domestic inputs ($\eta(s') > 1$).

[10] The time-varying markup $\mu_t(s)$ could be due either to price adjustment frictions, or variable demand elasticities. We consider versions of the model that incorporate these micro-foundations below, and just take the pricing rule in Equation 1 as given here.

Combining Equations 5, 6, and 8, we can write:

$$\hat{P}_{Ht}(s) = \hat{P}_{Vt}(s)^{1-\alpha(s)} \left[\prod_{s'} \hat{P}_{Ht}(s')^{\alpha(s',s)} \right] \left[\prod_{s'} \hat{\Lambda}^{M}_{Ht}(s',s)^{\alpha(s',s)/(\eta(s')-1)} \right]. \tag{9}$$

The first term captures changes in total factor productivity, markups, or primary factor costs. The second term captures changes in the prices of domestically produced inputs purchased by s from all upstream sectors, including sector s itself. The third term captures the impact of offshoring on unit costs.

Taking logs of Equation 9, and using lower case to denote the log of an upper case variable (i.e., $\hat{x}_t = ln\hat{X}_t$), yields:

$$\hat{p}_{Ht}(s) = (1-\alpha(s))\hat{p}_{Vt}(s) + \sum_{s'} \alpha(s',s)\hat{p}_{Ht}(s') + \sum_{s'} \left(\frac{\alpha(s',s)}{\eta(s')-1} \right) \hat{\lambda}^{M}_{Ht}(s',s), \tag{10}$$

This expression has input-output logic embedded in it, because the price of Home output in sector s depends on prices of output in all sectors at Home, including itself. The direct effect of a rise in foreign sourcing of inputs is to lower prices in sector s, and then this price reduction spills over across sectors, as sector s is used downstream as an input.

Stacking Equation 10 across sectors, we manipulate it to isolate domestic output prices:

$$\hat{\mathbf{p}}_{Ht} = \left[\mathbf{I} - \mathbf{A}'\right]^{-1} \left[\mathbf{I} - \alpha\right] \hat{\mathbf{p}}_{Vt} + \left[\mathbf{I} - \mathbf{A}'\right]^{-1} \left[\mathbf{A}' \circ \left(\hat{\lambda}^{M}_{Ht} \right)' \right] \left[\mathbf{H} - \mathbf{I}\right]^{-1} \iota, \tag{11}$$

where α is a matrix with $\alpha(s)$ along the diagonal and zeros elsewhere, $\mathbf{A}$ is an input-output matrix with elements $\alpha(s,s')$,$\hat{\lambda}^{M}_{Ht}$ is a matrix with elements $\hat{\lambda}^{M}_{Ht}(s,s')$, and $\mathbf{H}$ is a matrix with $\eta(s)$ along the diagonal. Further, $\iota$is a conformable column vector of ones, and $\circ$ denotes the Hadamard (entrywise) product of matrices. The first term is the downstream propagation of cost-push shocks to the price of real value added in all sectors. The second term is the downstream propagation of cost-push shocks attributable to offshoring, where an increase in domestic sourcing of inputs raises the price of domestic gross output. Intuitively, if inputs are increasingly sourced from home, we infer that that the price of imported inputs is rising relative to the price of domestic inputs, which implies that gross output prices will grow faster than implied by domestic value-added costs alone.

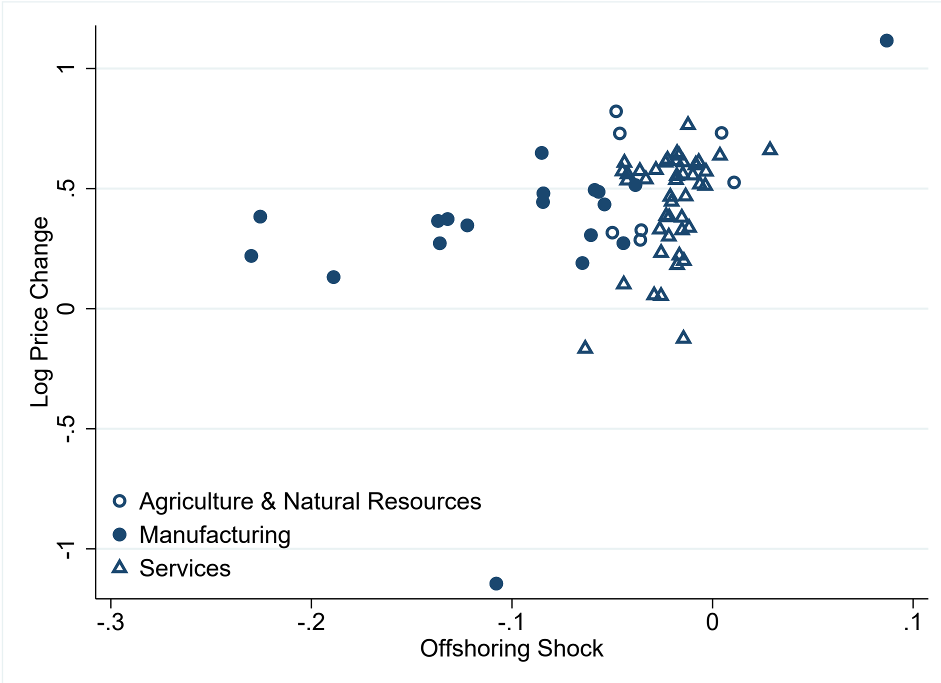
1.2 Data on Offshoring and Output Prices

We now turn to studying the impact of offshoring on domestic producer prices through the lens to Equation 11. We draw on two complementary data sets from the Industry Economic Accounts of the US Bureau of Economic Analysis (BEA). The first is a data set on the price of gross output by industry, from the GDP-by-industry statistics.[11] The second data set is the Input-Output Accounts, from which we construct annual input-output tables and domestic sourcing shares.[12] Both data sets include annual data for 1997-2018 for 71 summary-level industries, of which 26 are goods producing industries.

[11]These data are constructed primarily from PPI and CPI data collected by the BLS, and they are used by the BEA to deflate gross output in order to compute real GDP by industry. See BEA Gross-Domestic-Product-(GDP)-by-Industry Data. A useful point to emphasize that neither CPI data, nor PPI data, have appropriate product coverage to match up to industry-level data sources. The BEA combines various data sources in order to generate prices series that match the industry definitions and national accounts coverage in BEA data.

[12]We use make and use tables (after redefinitions, at producer value) and import use matrices (after redefinitions) to form industry-by-industry input-output tables split by domestic and imported input use, and to measure domestic and foreign sourcing of final goods. See BEA Input-Output Accounts Data.

Figure 1: Industry-Level Price Changes vs. the Offshoring Shock from 1997-2018



Note: Agriculture and Natural Resources includes BEA summary-level industries 1-7, Manufacturing includes 8-26, and Services includes 27-71. Each observation is the log difference between sector-level prices in 2018 and 1997: $\hat{p}_{Ht}(s) = \ln p_{H,2018}(s) - \ln p_{H,1997}(s)$.

From these data, we construct data analogs to $\hat{\mathbf{p}}_{Ht}$, $\mathbf{A}$ and $\hat{\lambda}^M_{Ht}$ in Equation 11. We measure $\hat{\mathbf{p}}_{Ht}$ using the log of the ratio of gross output price deflators in year t relative to 1997 in each industry. We take averages of the annual input-output matrices to form the time-invariant industry-to-industry input-output matrix: $\mathbf{A} \equiv (1/20)\sum_{t=1997}^{2018} \mathbf{A}_t$, where $\mathbf{A}_t = \mathbf{A}_{Ht} + \mathbf{A}_{Ft}$ is the total direct requirements matrix for year t, $\mathbf{A}_{Ht}$ is the domestic requirements matrix, and $\mathbf{A}_{Ft}$ is the import requirements matrix.[13] We then define domestic sourcing shares $\mathbf{\Lambda}^M_{Ht} \equiv \mathbf{A}_{Ht} \oslash \mathbf{A}_t$, and we construct ratios $\hat{\mathbf{\Lambda}}^M_{Ht} = \mathbf{\Lambda}^M_{Ht} \oslash \mathbf{\Lambda}^M_{H,1997}$, where $\oslash$ represents Hadamard (elementwise) division.

As a first pass, we assume that elasticities of substitution between home and foreign goods are equal in all sectors: $\eta(s) = \eta$. This implies that we can rewrite Equation 11 as:

$$\hat{\mathbf{p}}_{Ht} = [\mathbf{I} - \mathbf{A}']^{-1}\alpha\hat{\mathbf{p}}^v_t + \left(\frac{1}{\eta - 1}\right)\underbrace{[\mathbf{I} - \mathbf{A}']^{-1}\left[\mathbf{A}' \circ \left(\hat{\lambda}^M_{Ht}\right)'\right]\iota}_{\text{Offshoring Shock}}. \tag{12}$$

Each element of the term labeled Offshoring Shock is the cumulative impact of upstream offshoring on unit costs, accounting for both direct and indirect effects via the input-output structure. We can then relate price changes over a given horizon $\hat{\mathbf{p}}_{Ht}$ to the change in unit costs attributable to offshoring.

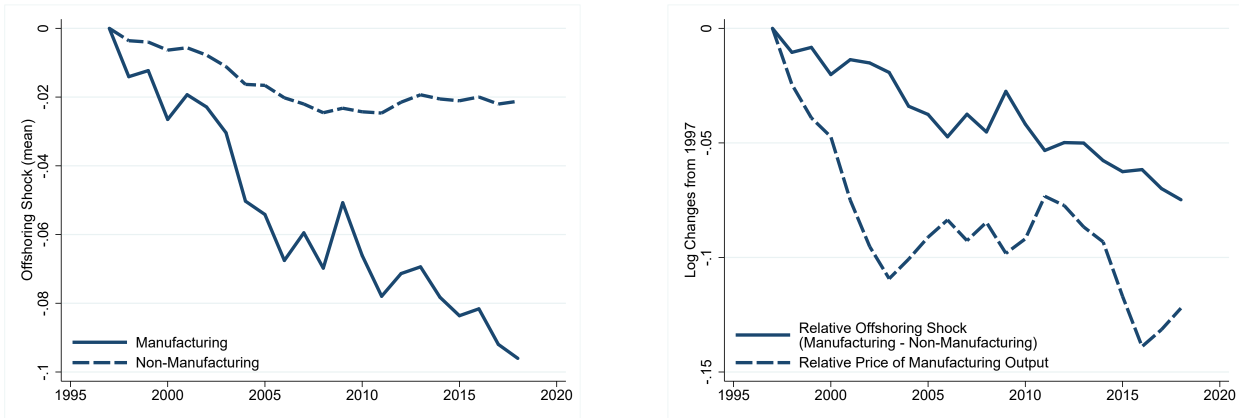
We plot long-run changes in $\hat{\mathbf{p}}_{Ht}$ versus the Offshoring Shock at the industry level in Figure 1. There is a positive correlation between the offshoring shock and price changes in the figure – industries with high

[13] The (i, j) elements in these matrices are the value of inputs purchased by industry j from industry i as a share of total gross output in industry j. $\mathbf{A}_{Ht}$ records purchases from domestic sources, while $\mathbf{A}_{Ft}$ records purchases from international sources. Note that we suppress changes in this domestic requirements matrix over time in the model, and thus in our empirical analysis as well.

Figure 2: Price Changes and the Offshoring Shock from 1997-2018

(a) Offshoring Shock in Manufacturing vs. Non-Manufacturing

(b) Relative Price and Offshoring Shock for Manufacturing vs. Non-Manufacturing



Note: The relative price of manufacturing in each year is $\frac{1}{|M|}\sum_{s\in M}\hat{\mathbf{p}}_{Ht}(s) - \frac{1}{|N|}\sum_{s\in N}\hat{\mathbf{p}}_{Ht}(s)$, where M and N denote the set of manufacturing and non-manufacturing industries, respectively.

exposure to offshoring, and thus the largest declines in direct and indirect home sourcing shares, experienced smaller output price changes. One point that is clear in the figure is that this correlation is driven by two aspects of the data: (a) there is a tight correlation within manufacturing between offshoring exposure and price changes, and (b) there are differences in the evolution of offshoring and prices across composite industry groups (e.g., manufacturing vs. agriculture, natural resources, and services). In contrast, there is only a weak relationship between the offshoring shock and price changes within non-manufacturing industries, in part because variation in offshoring changes within non-manufacturing industries are small.

To illustrate the time path of offshoring and price changes, we take simple averages of $\hat{\mathbf{p}}_{Ht}$ and the Offshoring Shock for manufacturing and non-manufacturing industries in each year. We plot the time series for these average offshoring shocks in manufacturing and non-manufacturing in Figure 3a. Manufacturing industries are impacted more than are Non-Manufacturing industries by offshoring, nearly five times as intensively. Turning to Figure 3b, we plot average log changes in the prices for manufactured manufacturing industries minus the same for non-manufacturing; More simply, this is an average relative price of manufacturing output. We also include the gap between the offshoring shock that hits the manufacturing and non-manufacturing sectors, which captures the relative offshoring shock across the two sectors. The relative price of manufacturing output declines over the period, coincident with the large relative offshoring shock that hit manufacturing industries.

Together, these data point to a role for offshoring in explaining output price changes over time. To put these in macro-context, we now turn to the evolution of consumer prices.

1.3 Final Goods and Aggregate Consumer Prices

The representative Home consumer has nested, constant elasticity of substitution (CES) preferences. We assume that she has Cobb-Douglas preferences across composite industry-level final goods, and that industry-level final goods are themselves CES composites of Home and Foreign final goods. The aggregate consumer price level is then given by:

$$P_{Ct} = \prod_s P_{Ct}(s)^{\gamma(s)} \tag{13}$$

$$\text{with} \quad P_{Ct}(s) = \Big(\nu(s) P_{Ht}(s)^{1-\eta(s)} + (1-\nu(s))\left(\tau_{Ct}(s) P_{Ft}(s)\right)\Big)^{1/(1-\eta(s))}, \tag{14}$$

where $P_{Ct}(s)$ is the price of a composite consumption good for industry s and $\tau_{Ct}(s)$ is an iceberg-type trade cost paid on imports of final goods.

Similar to the sufficient statistics argument for input use, we use first order conditions for the purchases of domestic consumption goods to write the expenditure share on domestic goods as: $\Lambda^C_{Ht}(s) = \frac{P_{Ht}(s)C_{Ht}(s)}{P_{Ct}(s)C_t(s)} = \nu(s)\left(\frac{P_{it}(s)}{P_{Ct}(s)}\right)^{1-\eta(s)}$. Taking ratios across time yields:

$$\hat{P}_{Ct}(s) = \hat{P}_{Ht}(s)\hat{\Lambda}^C_{Ht}(s)^{1/(\eta(s)-1)}. \tag{15}$$

If $\eta(s) > 1$, this says that aggregate consumer prices decline relative to the price of domestically produced goods when the share of spending on domestic goods falls over time. That is, $\hat{\Lambda}^C_{Ht}(s) < 1$ implies $\hat{P}_{Ct}(s) < \hat{P}_{Ht}(s)$.

The ratio of aggregate consumer prices in period t relative to the base period is: $\hat{P}_{Ct} = \prod_s \hat{P}_{Ct}(s)^{\gamma(s)}$. We combine this with Equation 15 and take logs to obtain:

$$\begin{aligned} \hat{p}_{Ct} &= \sum_s \gamma(s)\left[\hat{p}_{Ht}(s) + \left(\frac{1}{\eta(s)-1}\right)\hat{\lambda}^C_{Ht}(s)\right] \\ &= \gamma \hat{p}_{Ht} + \gamma\left[\mathbf{H} - \mathbf{I}\right]^{-1}\hat{\lambda}^C_{Ht}, \end{aligned} \tag{16}$$

where γ is a row vector with elements $\gamma(s)$, η is a diagonal matrix with elements $\eta(s)$, and $\hat{\lambda}^C_{Ht}$ is a column vector with elements $\hat{\lambda}^C_{Ht}(s)$. The second term in this expression captures the idea that falling prices for imported final goods– e.g., finished manufactured goods from China, like iPhones, clothing, shoes, or laptops – has restrained consumer price growth. This mechanism is the conventional channel featured in prior analyses of globalization and inflation.

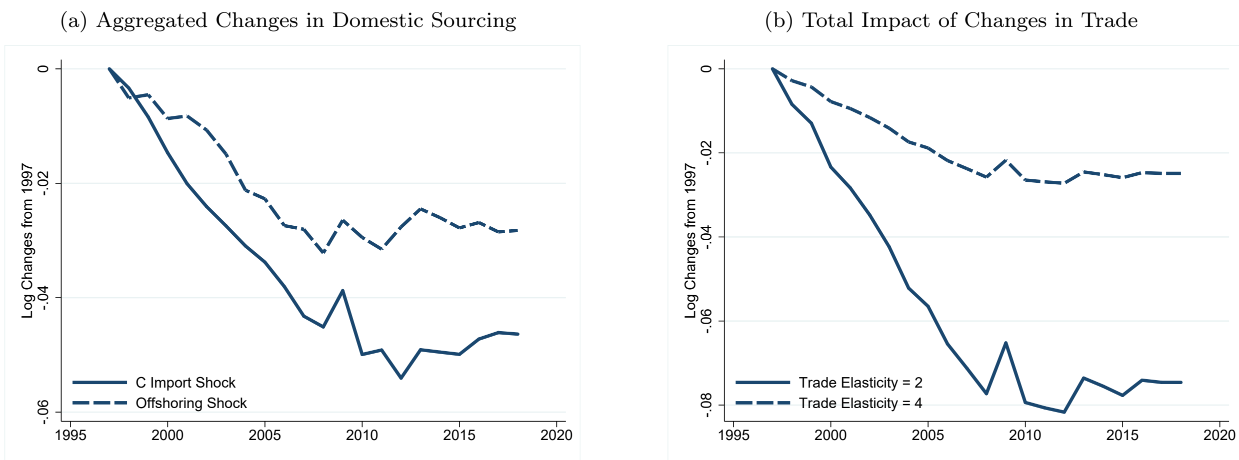
To this, our model adds an additional link between foreign sourcing and consumer prices. Specifically, we can insert Equation 12 into Equation 16 to decompose domestic prices:

$$\hat{p}_{Ct} = \gamma\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\alpha_i \hat{\mathbf{p}}^v_t + \underbrace{\left(\frac{1}{\eta-1}\right)\gamma\left[\mathbf{I} - \mathbf{A}'\right]^{-1}\left[\mathbf{A}' \circ \left(\hat{\lambda}^M_{Ht}\right)'\right]\iota}_{\text{Offshoring}} + \underbrace{\left(\frac{1}{\eta-1}\right)\gamma\hat{\lambda}^C_{Ht}}_{\text{C Imports}}, \tag{17}$$

where we have imposed $\eta(s) = \eta$ to simplify the expression. The first term is a measure of the dependence of consumer prices on domestic cost growth – changes in the the productivity-adjusted prices of domestic factors, i.e., domestic real value added. The second term is an adjustment for the impact of offshoring on consumer prices, which works through the impact that offshoring has in lowering prices for domestically produced goods. The third term captures the role of changes in trade in final goods on consumer prices. Note in both these terms, the formula tells us to aggregate sector-level changes in sourcing using sector expenditure shares in final demand, collected in γ.

We now turn to data to quantify the aggregate roles for offshoring and consumption imports in accounting for consumer prices. Using input-output data, we construct consumption of domestic goods as total personal

Figure 3: The Role of Offshoring and Consumption Imports in Accounting for Consumer Price Changes from 1997-2018



Note: In Panel (a), "C Import Shock" is $\gamma\hat{\lambda}^C_{Ht}$ and "Offshoring Shock" is $\gamma\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{A}'\circ\left(\hat{\lambda}^M_{Ht}\right)'\right]\iota$. In Panel (b), the total impact of changes in trade is $\left(\frac{1}{\eta-1}\right)\gamma\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{A}'\circ\left(\hat{\lambda}^M_{Ht}\right)'\right]\iota+\left(\frac{1}{\eta-1}\right)\gamma\hat{\lambda}^C_{Ht}$.

consumption expenditures (from the use table) less personal consumption expenditures reported in the import use table. From this, we compute Λ^C_{Ht} as the ratio of consumption of domestic goods to total consumption expenditure (as elsewhere, $\hat{\lambda}^C_{Ht}=\ln(\Lambda^C_{Ht}/\Lambda^C_{H,1997})$), and we compute the time-average share of consumption expenditure allocated to each sector, encoded in γ. To aggregate changes in offshoring, we use γ together with the input output matrix $\mathbf{A}$, defined previously. We plot the aggregate "shocks" – aggregated changes in domestic sourcing, given by and $\gamma\hat{\lambda}^C_{Ht}$ and $\gamma\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{A}'\circ\left(\hat{\lambda}^M_{Ht}\right)'\right]\iota$ – in Figure 3a. As is evident, both "shocks" are negative, consistent with falling domestic sourcing in the aggregate over time. Further, most of the decline is concentrated in the first half of the sample period (before 2010), and it is phased in slowly over time.[14]

To compute the impact of these changes on consumer prices requires taking a stand on the (matrix) value of η, the industry-level elasticities between home and foreign output. Estimating these separately for the 71 industries is beyond the scope of the exercise we want to perform here, so we impose a homogeneous elasticity ($\eta=\eta\mathbf{I}$). We then consider two alternative values for illustration: $\eta=\{2,4\}$.[15] We plot the results – i.e., the composite Offshoring and C Imports terms in Equation 17 – in Figure 3b. The cumulative impact of declines in domestic sourcing is to lower consumer prices relative to domestic value-added prices by between 2 and 8 percent over the sample period, depending on the elasticity. Translated into annual effects, rising trade lowers consumer price growth by between 10 and 40 basis points per year relative to growth in value-added prices. Increases in offshoring account for about 40% of this gap, while foreign sourcing of consumer goods accounts for the remainder.

[14]To head off possible confusion later, we note that this aggregation focuses entirely on consumption goods. In it, declines in domestic sourcing of consumption goods are more important than changes in offshoring in determining aggregate consumer prices. In the model below, we will combine sourcing of consumption and investment goods in calibrating shocks, which conforms to standard practice for models without physical capital in the literature.

[15]The value $\eta=2$ is near standard values of the Armington trade elasticity in the international macroeconomics literature. It is also close to a naive estimate of η obtained from the slope of a regression line through the scatter plot in Figure 1, as well as recent estimates of long run macro-elasticities based on tariff changes in Boehm, Levchenko and Pandalai-Nayar (2020). In the trade literature, $\eta=4$ is in the vicinity of standard values for estimated gravity trade elasticities (see Simonovska and Waugh (2014) for example). In calibrated models below, we set the elasticity between home and foreign goods equal to 3, which is the midpoint between these bounds.

1.4 Beyond Accounting

These results suggest that offshoring plays an important quantitative role in explaining the evolution of domestic prices across industries, and further that rising trade has lowered the aggregate consumer price level (depressed inflation). We now advance a word of caution about this second conclusion – that rising trade lowers the level of consumer prices – which motivates the model-based exercises that follow.

All the discussion of the aggregate consumer price level above is based on accounting decompositions. While it is tempting to interpret these decompositions as saying that offshoring lowered the consumer price level (i.e., lowered consumer price inflation), it is not possible to reach this conclusion from the accounting decomposition alone. In the aggregate, the evolution of the consumer price level is determined jointly by real factors (e.g., trade) and the reaction of monetary policy to them. Thus, we need a full-fledged model in which we specify how monetary policy reacts in order to make causal statements about how changes in trade – specifically, the exogenous driving forces that underlie those changes in trade – influence the evolution of the price level.

Notwithstanding this concern, we also emphasize that the earlier results about price changes across industries do speak to an important potential mechanism linking offshoring and prices. These results relate differences in price changes across sectors to differences in offshoring intensity, as in partial equilibrium differences-in-differences type analysis. If we treat differences in offshoring intensity as exogenous, then we could conclude that sectors with higher offshoring intensity have lower price growth. While this analysis does not identity the aggregate effects of offshoring, it does point to a mechanism that may give rise to aggregate price level effects. Thus, we turn to a full general equilibrium model in which we can evaluate the impact of trade in both inputs and final goods on inflation.

2 Baseline New Keynesian Model

This section develops a baseline New Keynesian model with trade in both inputs and final goods. In contrast to the multi-sector framework in Section 1, we focus on a one sector model in this section, which serves to isolate the distinct roles for final goods and intermediate inputs in the model and highlight the importance of trade dynamics for inflation outcomes. In particular, we are able to collapse this one sector model into a transparent "three equation model" – consisting of a Phillips curve, IS equation, and monetary policy rule – that yields sharp analytic results and crystallizes intuition. Having developed the baseline model, we then use it to explore the role of shocks to financial inflows and variable markups in shaping inflation outcomes in Section 3. We then return to a multisector version of the model in Section 4.

To outline this section, we first present the baseline model and illustrate how we link changes in domestic sourcing for inputs and final goods to inflation in the model. We then present model simulations to establish a "puzzling" result: according to the model, trade integration drove inflation up. To interpret this result, we then turn to the three equation representation of the model. We conclude by discussing the relationship between our results and "conventional wisdom" regarding the impact of trade on inflation.

2.1 The model

The model draws on the standard small open economy New Keynesian structure, as exposited by Galí (2015). We deviate from the textbook model by replacing Calvo-style pricing with Rotemberg pricing, which has no first order consequences for the questions we address (but accelerates exposition of the model). More importantly, we alter the production structure of the textbook model, allowing for trade in both final goods and inputs.[16] Further, we develop a new sufficient statistics approach to model analysis.

2.1.1 Consumers

Consumer preferences over labor supply L_t and consumption C_t are represented by:

$$U\left(\{C_t, L_t\}_{t=0}^{\infty}\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \mu \frac{L_t^{1+\psi}}{1+\psi}\right] \tag{18}$$

$$C_t = \left(\nu^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + (1-\nu)^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)} \tag{19}$$

$$C_{Ht} = \left(\int_0^1 C_{Ht}(i)^{(\epsilon-1)/\epsilon} di\right)^{\epsilon/(\epsilon-1)}, \tag{20}$$

where C_{Ft} is a composite foreign consumption good. The elasticity $\epsilon > 1$ controls substitution among domestic varieties, while η controls substitution between domestic and foreign goods. The parameter $\nu \in (0, 1)$ controls relative demand for home consumption goods, conditional on prices. The parameters $\rho \geq 0$ and $\psi > 0$ govern intertemporal substitution and labor supply in standard ways.

We assume that financial markets are complete, such that the consumer has access to a complete set of Arrow-Debreu securities that are traded internationally. The representative consumer faces the following budget constraint:

$$\int_0^1 P_{Ht}(i) C_{Ht}(i) di + P_{Ft} \tau_{Ct} C_{Ft} + E_t \left[Q_{t,t+1} B_{t+1}\right] \leq B_t + W_t L_t, \tag{21}$$

where B_t is the nominal, domestic currency payoff in period t of the portfolio of assets held by the consumer and $Q_{t,t+1}$ is the stochastic discount factor for nominal payments. The price of the foreign consumption good in domestic currency is P_{Ft}, and $\tau_{Ct} > 1$ is an iceberg-type trade cost paid on consumption imports. The prices of individual domestic goods are$\{P_{Ht}(i)\}$ and the nominal wage is W_t.

Given prices $\{\{P_{Ht}(i)\}, P_{Ft}, Q_{t,t+1}, W_t\}$ and initial asset holdings B_0, the consumer chooses consumption $\{C_t, \{C_{Ht}(i)\}, C_{Ft}\}$, labor supply $\{L_t\}$,and asset holdings $\{B_{t+1}\}$ to maximize Equations 18-20 subject to 21 and the standard transversality condition.

2.1.2 Production

The production function for individual domestic varieties is:

$$Y_t(i) = Z_t \left(L_t(i)\right)^{1-\alpha} \left(M_t(i)\right)^{\alpha} \tag{22}$$

$$M_t(i) = \left[\xi^{1/\eta} M_{Ht}(i)^{(\eta-1)/\eta} + (1-\xi)^{1/\eta} M_{Ft}(i)^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \tag{23}$$

$$M_{Ht}(i) = \left(\int_0^1 M_{Ht}(j,i)^{(\varepsilon-1)/\epsilon} dj\right)^{\epsilon/(\epsilon-1)}, \tag{24}$$

where $L_t(i)$ and $M_t(i)$ are quantities of labor and a composite input used by firm i. The composite input is a nested CES composite of inputs sourced from home and abroad: $M_{Ht}(j, i)$ is the quantity of inputs

[16] We allow for different levels and changes in foreign sourcing of inputs and final goods, unlike Gopinath et al. (2020), who also allow for input trade.

from Home firm j purchased by firm i, $M_{Ht}(i)$ is the composite home input used by firm i, and $M_{Ft}(i)$ is the quantity of a foreign composite input purchased by firm i. Similar to consumption, $\epsilon > 1$ controls substitution among domestic varieties, while η controls substitution across country sources for inputs. The parameter $\xi \in (0, 1)$ controls relative demand for home inputs, conditional on prices.

Producers of differentiated output set the prices of their goods under monopolistic competition, and they select the input mix to satisfy the implied demand. These two problems can be analyzed separately.

Pricing Each Home firm sets its price in domestic currency, which applies to both output sold domestically and exports.[17] It chooses a sequence for $P_{Ht}(i)$ to maximize the present discounted value of profits, inclusive of quadratic adjustment costs incurred when it changes prices, as in Rotemberg (1982*a*,*b*). Letting $MC_t(i)$ be the firm's marginal cost of production (defined below), the present value of profits is:

$$E \sum_{t=0}^{\infty} \beta^t \frac{C_t^{-\rho}}{C_0^{-\rho}} \frac{1}{P_{Ct}} \left[P_{Ht}(i) Y_t(i) - MC_t(i) Y_t(i) - \frac{\phi}{2} \left(\frac{P_{Ht}(i)}{P_{H,t-1}(i)} - 1 \right)^2 P_{Ht} Y_t \right],$$

where the last term records the quadratic price adjustment costs. In this adjustment cost term, ϕ is a parameter that controls the degree of price rigidity, $Y_t = \int_0^1 Y_t(i) di$ is total home output, and $P_{Ht} = \left(\int_0^1 P_{Ht}(i)^{(1-\epsilon)/\epsilon} di \right)^{1/(1-\epsilon)}$ is the price of the CES bundle of home output.

Input Demand Firm i in sector s chooses $\{L_t(i), M_t(i), M_{Ht}(i), M_{Ft}(i), M_{Ht}(j, i)\}$ to minimize the cost of producing output $Y_t(i)$. Similar to consumers, firms pay iceberg trade costs τ_{Mt} on inputs they import from abroad. Thus, variable production costs are $W_t L_t(i) + P_{Mt} M_t(i)$, with $P_{Mt} M_t(i) = P_{Ht} M_{Ht}(i) + \tau_{Mt} P_{Ft} M_{Ft}(i)$ and $P_{Ht} M_{Ht}(i) = \int_0^1 P_{Ht}(j) M_{Ht}(j, i) dj$. Here $P_{Mt} = \left[\xi P_{Ht}^{1-\eta} + (1 - \xi) \left(\tau_{Mt} P_{Ft} \right)^{1-\eta} \right]^{1/(1-\eta)}$ is the price of the composite input.

2.1.3 Closing the Model

Demand for exports of individual domestic varieties $X_t(i)$ has a CES structure, such that demand for firm i's exports is given by:

$$X_t(i) = \left(\frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\epsilon} X_t, \tag{25}$$

$$\text{with} \quad X_t = \left(\frac{P_{Ht}}{S_t P_{Ct}^*} \right)^{-\eta} C_t^*, \tag{26}$$

where S_t is the nominal exchange rate (units of domestic currency to buy 1 unit of foreign currency), P_{Ct}^* is the foreign price index in foreign currency, and C_t^* is foreign consumption.

The market clearing conditions for output of each variety and the labor market are:

$$Y_t(i) = C_{Ht}(i) + \int_0^1 M_{Ht}(i, j) dj + X_t(i) + \frac{\phi}{2} \left(\frac{P_{Ht}(i)}{P_{H,t-1}(i)} - 1 \right)^2 Y_t \tag{27}$$

$$\int_0^1 L_t(i) = L_t. \tag{28}$$

[17] This producer currency pricing assumption for exports is consistent with the body of evidence on dollar pricing of US exports [Gopinath et al. (2020)].

Further, due to trade in the stage contingent asset, the usual international risk sharing condition applies:

$$\left(\frac{C_t}{C_t^*}\right)^{-\rho}\left(\frac{S_t P_{Ct}^*}{P_{Ct}}\right) = \Xi, \tag{29}$$

where Ξ is a constant, which depends on initial conditions.[18]

Finally, we specify a Taylor-type monetary policy rule to close the model:

$$1 + i_t = (1 + i_0)\left(\frac{P_{Ct}}{P_{C,t-1}}\right)^{\omega}, \tag{30}$$

where we assume the central bank responds to consumer price inflation and i_0 is the steady state (date 0) interest rate.

2.1.4 Equilibrium

We define an equilibrium for the small open economy taking foreign variables as given, including the price of foreign goods in domestic currency.[19] As is standard, we focus on a symmetric equilibrium, where all domestic producers are identical, so drop the firm/variety index. Given exogenous variables $\{P_{Ft}, C_t^*, P_t^*, Z_t, \tau_{Ct}, \tau_{Mt}\}$, an equilibrium (up to a normalization) is a collection of prices $\{W_t, P_{Ht}, P_{Ct}, P_{Mt}, MC_t, S_t, i_t\}$ and quantities $\{C_t, C_{Ht}, C_{Ft}, L_t, X_t, Y_t, M_t, M_{Ht}, M_{Ft}\}$ that solve the consumer's utility maximization problem, the producer's pricing and input demand problems (maximize profits), and clear the markets for goods, labor, and state-contingent assets. Further, interest rates are set based on the monetary policy rule. For reference, we collect the symmetric equilibrium conditions in Table 1.

2.1.5 Equilibrium with Domestic Sourcing Shares as Sufficient Statistics

To analyze the model, we first redefine variables to highlight the role of trade openness in driving the results. Let $\Lambda_{Ht}^C \equiv \frac{P_{Ht}C_{Ht}}{P_{Ct}C_t}$ and $\Lambda_{Ht}^M = \frac{P_{Ht}M_{Ht}}{P_{Mt}M_t}$ be the shares of final and input expenditure that falls on home produced goods, which we refer to as the "domestic sourcing shares." We use the first order conditions describing demand for final goods and inputs to solve for the relative price of home goods:

$$\frac{P_{Ht}}{P_{Ct}} = \left(\frac{\Lambda_{Ht}^C}{\nu}\right)^{1/(1-\eta)} \tag{31}$$

$$\frac{P_{Ht}}{P_{Mt}} = \left(\frac{\Lambda_{Ht}^M}{\xi}\right)^{1/(1-\eta)}. \tag{32}$$

These equations say that we can use domestic sourcing shares, together with the value of the trade elasticity (η), to infer the price of home goods relative to the final goods and input bundles. Put differently, the domestic sourcing shares and the trade elasticity are sufficient statistics for these relative prices, as in Arkolakis, Costinot and Rodríguez-Clare (2012).

Using this result, we can substitute out for these relative prices throughout the equilibrium system in Table 1, and thus redefine the equilibrium for given values of the domestic sourcing shares. Using this

[18] Introducing additional notation, $\Xi = \frac{E_0\theta_0}{\theta_0^*}$, where E_0 is the date zero exchange rate, and θ_0 and θ_0^* are date zero Lagrange multipliers on lifetime budget constraints of home and foreign agents.

[19] Ordinarily, the domestic price of foreign goods (P_{Ft}) would be an equilibrium object in a small open economy model, and its behavior would depend on pricing assumptions. We treat it as exogenous here for brevity, since it will not be needed to define the equilibrium in terms of sourcing shares below. We discuss the redundancy of dollar currency import prices again when we model foreign price setting in Section 3.2 and Appendix E.

Table 1: Baseline Model Summary

Consumption-Leisure	$C_t^{-\rho}\frac{W_t}{P_{Ct}} = \mu L_t^{\psi}$
Consumption Allocation	$C_{Ht} = \nu\left(\frac{P_{Ht}}{P_{Ct}}\right)^{-\eta} C_t$
	$C_{Ft} = (1-\nu)\left(\frac{\tau_{Ct}P_{Ft}}{P_{Ct}}\right)^{-\eta} C_t$
Euler Equation	$1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_{Ct}}{P_{C,t+1}}(1+i_t)\right]$
Input Choices	$W_tL_t = (1-\alpha)MC_tY_t$
	$P_{Mt}M_t = \alpha MC_tY_t$
	$M_{Ht} = \xi\left(\frac{P_{Ht}}{P_{Mt}}\right)^{-\eta} M_t$
	$M_{Ft} = (1-\xi)\left(\frac{\tau_{Mt}P_{Ft}}{P_{Mt}}\right)^{-\eta} M_t$
Marginal Cost	$MC_t = \frac{W_t^{1-\alpha}P_{Mt}^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}Z_t}$
Price Setting	$(1-\epsilon)+\epsilon\frac{MC_t}{P_{Ht}} - \phi\left(\frac{P_{Ht}}{P_{H,t-1}}-1\right)\frac{P_{Ht}}{P_{H,t-1}}$
	$+E_t\left[\beta\frac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\frac{P_{Ct}}{P_{C,t+1}}\phi\left(\frac{P_{H,t+1}}{P_{H,t}}-1\right)\frac{P_{H,t+1}Y_{t+1}}{P_{Ht}Y_t}\frac{P_{H,t+1}}{P_{Ht}}\right]=0$
Price Indexes	$P_{Ct} = \left[\nu P_{Ht}^{1-\eta} + (1-\nu)\left(\tau_{Ct}P_{Ft}\right)^{1-\eta}\right]^{1/(1-\eta)}$
	$P_{Mt} = \left[\xi P_{Ht}^{1-\eta} + (1-\xi)\left(\tau_{Mt}P_{Ft}\right)^{1-\eta}\right]^{1/(1-\eta)}$
Market Clearing	$Y_t = C_{Ht} + M_{Ht} + X_t + \frac{\phi}{2}\left(\frac{P_{Ht}}{P_{H,t-1}}-1\right)^2 Y_t$
	$X_t = \left(\frac{P_{Ht}}{S_tP_{Ct}^{*}}\right)^{-\eta} C_t^{*}$
	$\left(\frac{C_t}{C_t^{*}}\right)^{-\rho}\left(\frac{S_tP_{Ct}^{*}}{P_{Ct}}\right) = \Upsilon$
Monetary Policy Rule	$1+i_t = (1+i_0)\left(\frac{P_{Ct}}{P_{C,t-1}}\right)^{\omega}$

Table 2: Log-Linearization of the Baseline Model

Consumption-Leisure	$\hat{l}_t = -\frac{\rho}{\psi}\hat{c}_t + \frac{1}{\psi}\widehat{rw}_t - \frac{1}{\psi(\eta-1)}\hat{\lambda}^C_{Ht}$
Consumption Allocation	$\hat{c}_{Ht} = \frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \hat{c}_t$
Euler Equation	$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\rho}\left(\hat{r}_t - E_t\pi_{Ct+1}\right)$
Input Choices	$\hat{l}_t = \widehat{rmc}_t + \hat{y}_t - \widehat{rw}_t$ $\hat{m}_t = \widehat{rmc}_t + \hat{y}_t - \frac{1}{\eta-1}\hat{\lambda}^M_{Ht}$ $\hat{m}_{Ht} = \frac{\eta}{\eta-1}\hat{\lambda}^M_{Ht} + \hat{m}_t$
Real Marginal Cost	$\widehat{rmc}_t = (1-\alpha)\widehat{rw}_t + \frac{\alpha}{\eta-1}\hat{\lambda}^M_{Ht} - \hat{z}_t$
Domestic Price Inflation	$\pi_{Ht} = \left(\dfrac{\epsilon-1}{\phi}\right)\widehat{rmc}_t + \beta E_t\left(\pi_{Ht+1}\right)$
Consumer Price Index	$\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)}\left(\hat{\lambda}^C_{Ht} - \hat{\lambda}^C_{Ht-1}\right)$
Market Clearing	$\hat{y}_t = \left(\frac{C_{H0}}{Y_0}\right)\hat{c}_{Ht} + \left(\frac{M_{H0}}{Y_0}\right)\hat{m}_{Ht} + \left(\frac{X_0}{Y_0}\right)\hat{x}_t$ $\hat{x}_t = \frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \eta\hat{q}_t + \hat{c}^*_t$ $\hat{c}_t = \hat{c}^*_t + \frac{1}{\rho}\hat{q}_t$
Monetary Policy Rule	$\hat{r}_t = \omega\pi_{Ct}$

method to reduce down the model, the log-linearized equilibrium conditions are presented in Table 2, where all variables are expressed as log deviations from steady state (i.e., $\hat{x}_t = \ln(X_t) - \ln(X_0)$ for variable X and the subscript 0 indexes an initial steady state). In the table, we define $\widehat{rmc}_t \equiv \widehat{mc}_t - \hat{p}_{Ht}$ to be real marginal costs and $\widehat{rw}_t \equiv \hat{w}_t - \hat{p}_{Ht}$ to be the real wage. Further, $\hat{r}_t \equiv \ln(1+i_t) - \ln(1+i_0) \approx i_t - i_0$, and $\hat{q}_t \equiv \hat{s}_t + \hat{p}^*_{Ct} - \hat{p}_{Ct}$ is the consumption real exchange rate. Finally, inflation rates are given by: $\pi_{Ht} = \hat{p}_{Ht} - \hat{p}_{H,t-1}$ and $\pi_{Ct} = \hat{p}_{Ct} - \hat{p}_{C,t-1}$.

We will analyze the dynamic equilibrium in this reduced model taking the path of domestic sourcing shares as given. Given domestic sourcing shares $\{\hat{\lambda}^C_{Ht}, \hat{\lambda}^M_{Ht}\}$ and exogenous shocks $\{\hat{z}_t, \hat{c}^*_t\}$, an equilibrium is a path for prices $\{\hat{q}_t, \hat{e}_t, \pi_{Ct}, \pi_{Ht}, \widehat{rw}_t, \widehat{rmc}_t, \hat{r}_t\}$ and quantities $\{\hat{c}_t, \hat{y}_t, \hat{l}_t, \hat{x}_t, \hat{c}_{Ht}, \hat{m}_{Ht}\}$ that satisfies the equilibrium conditions in Table 2.[20]

This equilibrium definition highlights the value of the sufficient statistics approach in the model. In this equilibrium, we need not directly track trade costs, or the price of foreign goods, over time. As a result, this method sidesteps a host of difficult data and theoretical issues. On the data side, we avoid needing to directly measure trade costs or foreign prices. Further, we need not make theoretical assumptions about currency invoicing or pass-through of foreign cost shocks into import prices. Instead, we lean on the model result that the domestic sourcing share – agents' responses to implicit price changes – tells us everything we need to know about relative international prices to study domestic equilibrium outcomes.[21]

[20] Note that given domestic sourcing shares $\{\hat{\lambda}^C_{Ht}, \hat{\lambda}^M_{Ht}\}$, we do not need to explicitly solve for imported consumption goods ($\hat{c}_{Ft}$) and inputs ($\hat{m}_{Ft}$), but these can be computed ex post using the equilibrium conditions if desired.

[21] To elaborate further, for historical simulations that take the path of past domestic sources shares as given by data, our approach is identical to the following alternative approach. One could specify an explicit model of import prices (e.g., producer currency pricing for imports, or dollar invoicing of imports). This would link the domestic currency price of imports to foreign marginal costs (in foreign currency) and trade costs. Conditional on foreign marginal costs and the elasticity of substitution between imports and domestic goods, one could then pick the value of trade costs to exactly match the observed share of imports in expenditure over time. Simulating the model with this imputed trade cost series exactly replicates simulation of the model taking the domestic source share as given, as in Table 2. Further, one would obtain identical outcomes for inflation and

Before pushing on to implement this approach, we pause to note one potential shortcoming of it. Specifically, it is best suited to studying the inflationary impacts of "external shocks" on inflation, such as changes in foreign prices (P_{Ft}) or trade costs (τ_{Ct}, τ_{Mt}), because these shocks influence inflation only through domestic sourcing shares. It is less well suited to analyze the inflationary impact of "domestic shocks" – e.g., a change in domestic productivity – that impact inflation directly (i.e., conditional on domestic sourcing shares) as well as indirectly, through their impact on domestic sourcing shares. To simulate the effects of this kind of shock, one would need to directly model domestic sourcing shares, which in turn requires assumptions about import pricing.

Given our focus on understanding the effects of rising trade integration on inflation, the sufficient statistics approach has benefits that outweigh this cost. First, it simplifies algebraic analysis of the model, which we exploit below. Second, in our empirical context, changes in foreign prices (e.g., Chinese productivity growth) and/or trade frictions (e.g., falling tariffs, increases in logistics efficiency, etc.) are likely the first order determinants of declines in domestic sourcing over time. Further, to the extent that domestic shocks have indirect effects via trade, these are included in our overall tally of the impact of changing trade on inflation under the sufficient statistics approach. Thus, we proceed to implementation.

2.2 Simulated Impact of Trade on Inflation

We apply the model to simulate consumer inflation given the observed evolution of aggregate domestic sourcing shares ($\hat{\lambda}^C_{Ht}$ and $\hat{\lambda}^M_{Ht}$) from 1997-2018. Assuming that the economy is in steady state prior to 1997, and that each time period corresponds to one quarter, we calibrate the model using standard external parameter values and the expenditure shares from the BEA data discussed in Section 1.[22] The parameter values are recorded in Table 3.[23] We introduce globalization as an MIT-style shock: starting from a static equilibrium, agents learn the sequence of shocks, and we solve for the dynamics that result from them under

other macro variables, regardless of the currency invoicing of imports. The currency invoicing of imports would only change the imputed trade cost series needed to match observed domestic sourcing shares. For prospective analysis of the future impacts of changes in current trade costs, currency invoicing matters in that it determines the mapping from policy changes to domestic sourcing shares.

[22]In the data, we assign all final goods imports to consumption, consistent with the absence of a government or investment sector in this baseline model. We construct a quarterly series for domestic sourcing shares from annualized data via interpolation. From annual BEA data, we have annual values for λ^C_{Ht} and λ^M_{Ht}, where $t \in \{1997, 2018\}$. We define domestic sourcing shares for the pre-1997 steady state as follows: $\lambda^U_{H0} = \max\left\{\lambda^U_{H1997} + (\lambda^U_{H1997} - \lambda^U_{H1998}), \lambda^U_{H1998}\right\}$, for $U = \{C, M\}$. This has the following interpretation. If λ^U_H declines from 1997 to 1998, we assume that the reduction from 1996 to 1997 is the same as from 1997 to 1998. If instead, λ^U_H does not decline from 1997 to 1998, we assume that the value λ^U_H in 1996 is the same as in 1998. This particular formulation captures the notion that home shares tend to decline, but in some cases, they oscillate a bit during the late 90s before they start declining. In the light of these patterns, we set the pre-1997 state as slightly more closed than 1997 in the first scenario, and equal to the state we observe in 1998 in the second scenario. Our results do not hinge on this particular set up, as we have experimented with various approaches. After defining the steady state levels, we compute log deviations of the annual home shares from steady state ($\hat{\lambda}^C_{Ht}, \hat{\lambda}^M_{Ht}$). We then conduct quarterly interpolation as follows: $\hat{\lambda}^U_{Ht.Q1} = 0.4\hat{\lambda}^U_{Ht-1} + 0.6\hat{\lambda}^U_{Ht}$, $\hat{\lambda}^U_{Xt.Q2} = 0.2\hat{\lambda}^U_{Ht-1} + 0.8\hat{\lambda}^U_{Ht}$, $\hat{\lambda}_{Xt.Q3} = 0.8\hat{\lambda}^U_{Ht} + 0.2\hat{\lambda}^U_{Ht+1}$, $\hat{\lambda}_{Xt.Q4} = 0.6\hat{\lambda}^U_{Ht} + 0.4\hat{\lambda}^U_{Ht+1}$, where subscript $t.Qx$ denotes the value for year t and quarter x. Henceforth in the paper, t indexes quarters in the model and simulations.

[23]In this parameterization, we set the elasticity of substitution among domestic varieties (ϵ) equal to the elasticity of substitution between home and foreign goods (η). Put differently, we set the macro-Armington elasticity between home and foreign goods equal to the microeconomic elasticity between domestic varieties. This is consistent with estimates in Feenstra et al. (2018), which fails to reject equality of these elasticities for most goods. This parameterization also facilitates comparison of this baseline model to the model with variable markups to be presented later. Further, relaxing it does not qualitatively change our results. As discussed above, the elasticity of substitution between home and foreign goods is set in the middle of the range of available estimates in the macroeconomics and trade literatures. Regarding price rigidity, Sims and Wolff (2017) provides the equivalence formula between the parameter governing price rigidity in Rotemberg versus Calvo-style models: $\phi = \frac{\kappa(\epsilon-1)}{(1-\kappa)(1-\beta\kappa)}$, where $1-\kappa$ is the share of firms that adjust their prices each period in a Calvo-style model. We set $\kappa = .75$, to match the average duration of prices, which leads to the value for ϕ in the table, given other parameters.

Table 3: Calibration

Parameter	Value	Reference/Target
ψ	2	Labor supply elasticity of 0.5
β	.995	Annual risk-free real rate of 2%
ϵ	3	Elasticity of substitution between home varieties
η	3	Elasticity of substitution between home and foreign goods
ρ	2	Intertemporal elasticity of substitution of 0.5
α	.434	To match 1996 input share
ν	.96	To match 1996 home share in consumption
ξ	.925	To match 1996 home share in intermediates
ϕ	23.6453	To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters [Sims and Wolff (2017)].
ω	1.5	Clarida, Gali and Gertler (1999)
Υ	0.008	To match the trade deficit to gross output ratio in 1996, given by $\frac{P_{F0}(\tau_{C0}C_{F0}+\tau_{M0}M_{F0})-P_{H0}X_0}{P_{H0}Y_0} = 0.0073$.

perfect foresight.[24]

Figure 4a plots the evolution of the domestic sourcing shocks $\hat{\lambda}^C_{Ht}$and $\hat{\lambda}^M_{Ht}$. Domestic sourcing falls for both final goods and inputs, with changes that are phased in over time. Moreover, medium term dynamics feature prominently: the input sourcing share follows a u-shaped pattern, and there are sharp adjustments in both sourcing shares around the Great Recession in 2008-2011.

Given these shocks, we plot simulated consumer price inflation (π_{Ct}) in Figure 4b. The solid line records inflation when domestic sourcing for both final goods and inputs changes in the model, while the dashed lines record simulated inflation for each shock separately (i.e., fed into the model one at a time). With both shocks active, inflation doesn't change much at the outset (1997-2000), but then rises by about 75 basis points after the year 2000. It remains positive for the remainder of the 2000-2010 interval, and only falls below zero for a sustained period after 2010. Adding up these changes over time, the price level rises by about 17% between 1997 and 2010. It then falls thereafter, but the post-2010 deflation doesn't make up for the pre-2010 inflation – the price level stabilizes at a level about 8% higher than its 1997 level as a result of the changes in domestic sourcing. Thus, changes in trade led to net inflation over this time period, averaging 40 basis points per year.

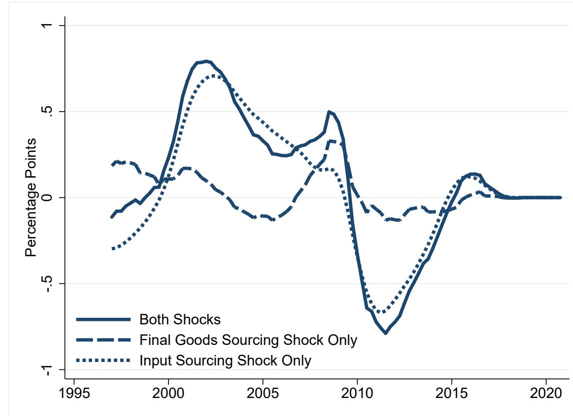
The path of inflation reflects the impact of changes in final goods and input sourcing. Both contribute to high inflation in the early 2000's, though the role of inputs is far larger. Further, the post-2010 deflation is driven primarily by the dynamics of input sourcing. Referring to the shocks, domestic sourcing is declining during the 2000-2010 period, coincident with positive inflation in the model. In contrast, as domestic sourcing for inputs rises after 2010, inflation is negative. Thus, this simulation suggests that inflation comoves negatively with domestic sourcing: periods of increased globalization are associated with inflation,

[24] As a terminal condition for the shocks, we assume that agents expect the level of domestic sourcing that prevails in 2018 to persist into perpetuity thereafter. Altering this assumption impacts the last few years of the simulation (changes are modest in size), but has deminimus effects on outcomes in early periods.

Figure 4: Shocks and Simulation of Inflation in the Baseline Model

(a) Changes in Domestic Sourcing

(b) Simulated Inflation



while retrenchment leads to deflation.

These results are puzzling on the surface, and contradict the conventional wisdom about the impact of trade integration on inflation. Having established this trade and inflation puzzle, we turn back to the model to present theoretical analysis that illuminates the role of trade dynamics in driving inflation.

2.3 The Three Equation Model

To provide intuition regarding the impact of trade on inflation, we distill the equilibrium system in Table 2 into a three equation model, with a Phillips curve, IS curve, and monetary policy rule.

2.3.1 Phillips Curve

We start by deriving a Phillips curve for consumer prices, linking aggregate inflation to a suitably defined output gap. As a first step, we derive the Phillips curve for domestic output prices.

In Appendix B, we show that real marginal costs depend on the gap between real wages in the actual and flexible price equilibria: $\widehat{rmc}_t = (1-\alpha)\left[\widehat{rw}_t - \widehat{rw}^n_t\right]$, where superscript n defines the value of a variable under flexible prices. The real wage gap can then be linked to the output, through goods and labor market clearing: $\widehat{rw}_t - \widehat{rw}^n_t = \chi\left[\hat{y}_t - \hat{y}^n_t\right]$, where $\hat{y}^n_t$ is the log deviation of gross output in a flexible price equilibrium from its initial steady state value, and $\chi > 0$ is function of fundamental parameters and steady state expenditure shares.[25]

Inserting these results into the equation for domestic price inflation gives us the relationship between domestic price inflation and the output gap, which we refer to as the domestic price Phillips curve:

$$\pi_{Ht} = \left(\frac{(\epsilon-1)(1-\alpha)\chi}{\phi}\right)\left[\hat{y}_t - \hat{y}^n_t\right] + \beta E_t\left(\pi_{Ht+1}\right), \tag{33}$$

with $\frac{(\epsilon-1)(1-\alpha)\chi}{\phi} > 0$, so domestic price inflation is increasing in the domestic output gap.

An important point to note here that changes in domestic sourcing have no direct impact on domestic price inflation, conditional on the output gap. This result may seem counterintuitive, because increased

[25] While we write the output gap in terms of gross output here, we show the model can be rewritten in terms of the output gap for real value added in Appendix B.

offshoring lowers domestic production costs. To elaborate, note that domestic price inflation is given by $\pi_{Ht} = \left(\frac{\epsilon-1}{\phi}\right)\widehat{rmc}_t + \beta E_t\left(\pi_{Ht+1}\right)$, with $\widehat{rmc}_t = (1-\alpha)\widehat{rw}_t + \frac{\alpha}{\eta-1}\hat{\lambda}^M_{Ht} - \hat{z}_t$. At first glance, it looks like decline in domestic sourcing of inputs ($\hat{\lambda}^M_{Ht} < 0$) lowers real marginal costs, and thus lower real marginal costs should lower domestic inflation. The reason this logic is misleading is that it conditions on the real wage ($\widehat{rw}_t$), which is an endogenous object. The domestic price Phillips curve accounts for changes in the real wage through the output gap. Further, by focusing attention on the gap between the actual and flexible price equilibria, this analytical approach nets out changes in domestic sourcing that influence both the actual and flexible price equilibria symmetrically.

Combining Equation 33 with the consumer price index, the consumer price Phillips curve is:

$$\pi_{Ct} = \left(\frac{(\epsilon-1)(1-\alpha)\chi}{\phi}\right)\left[\hat{y}_t - \hat{y}^n_t\right] + \beta E_t \pi_{Ct+1} + \frac{1}{(\eta-1)}\left[\Delta\hat{\lambda}^C_{Ht} - \beta E_t \Delta\hat{\lambda}^C_{Ht+1}\right], \tag{34}$$

where $\Delta\hat{\lambda}^C_{Ht} \equiv \hat{\lambda}^C_{Ht} - \hat{\lambda}^C_{Ht-1} = \Delta \ln \lambda^C_{Ht}$ is the log change in domestic sourcing across adjacent periods.

In contrast to domestic price inflation, changes in domestic sourcing for final goods directly impact consumer price inflation. Given expected future changes in domestic final goods sourcing ($E_t\Delta\hat{\lambda}^C_{Ht+1}$) and inflation ($\pi_{Ct+1}$), and the current output gap ($\hat{y}_t - \hat{y}^n_t$), a reduction in domestic sourcing today ($\Delta\hat{\lambda}^C_{Ht}<0$) lowers consumer price inflation. This is intuitive, as a reduction in domestic sourcing is associated with a terms of trade improvement – falling prices for imported relative to domestic consumption goods, which directly lowers overall consumer price inflation. In contrast, an anticipated reduction in domestic sourcing tomorrow ($E_t\Delta\hat{\lambda}^C_{Ht+1} < 0$) raises consumer price inflation today, all else equal. The reason is that, given π_{Ct+1}, lower domestic sourcing at date $t+1$ implies that future domestic price inflation π_{Ht+1} must be higher, and this higher future domestic price inflation raises domestic price inflation today, via Equation 33. These results emphasize that the dynamics of domestic sourcing are important for understanding inflation, and we will return to this point below.

2.3.2 IS Curve

The output gap reflects the structure of the Euler equation, as is standard. Referring again to Appendix B for detailed derivation, the IS curve can be written as:

$$\left[\hat{y}_t - \hat{y}^n_t\right] = -\frac{1}{\theta\rho}\left[\hat{\tilde{r}}_t - \hat{\tilde{r}}^n_t\right] + E_t\left[\hat{y}_{t+1} - \hat{y}^n_{t+1}\right], \tag{35}$$

where $\hat{\tilde{r}}_t \equiv \hat{r}_t - E_t\pi_{Ct+1}$ is the real interest rate, $\hat{\tilde{r}}^n_t \equiv \hat{r}^n_t - E_t\pi^n_{Ct+1} = \hat{r}^n_t - \frac{1}{(\eta-1)}E_t\Delta\ln\lambda^C_{Ht+1}$ is the real interest rate in the flexible price equilibrium (the natural real interest rate), and $\theta>0$ depends on primitive parameters and steady state expenditure shares. To complete the characterization of the IS curve, we solve for the real interest rate in the flexible price equilibrium as a function of exogenous variables:

$$\hat{\tilde{r}}^n_t = \mathit{\Omega}_{C^*}E_t\Delta\hat{c}^*_{t+1} + \mathit{\Omega}_Z E_t\Delta\hat{z}_{t+1} + \mathit{\Omega}_M E_t\Delta\hat{\lambda}^M_{Ht+1} + \mathit{\Omega}_C E_t\Delta\hat{\lambda}^C_{Ht+1}, \tag{36}$$

where $\mathit{\Omega}_{C^*} > 0$, $\mathit{\Omega}_Z > 0$, $\mathit{\Omega}_M < 0$, and $\mathit{\Omega}_C < 0$ are functions of parameters and steady state values, defined in Appendix B.

The natural real interest rate depends on expected future changes in domestic sourcing for both final goods and inputs. It is higher when domestic sourcing is expected to decline in the future, equivalently when the terms of trade are expected to improve. There are two distinct channels at work. First, domestic sourcing of final goods directly matters for the path of consumption. An expected terms of trade improvement for consumer goods means that consumers have higher real income in the future, holding their nominal income constant. This leads them to attempt to pull consumption forward, which drives up the flexible price equilibrium interest rate ($\hat{r}_t^n$) today. Second, domestic sourcing of inputs matters for consumer income via its impacts on the production side of the economy: an improvement in the terms of trade for sourcing inputs has similar effects to an increase in productivity. By lowering future production costs, an expected decline in domestic sourcing of inputs leads to higher future output and income. This increase in future income further leads consumers to attempt to pull consumption forward in time, leading the natural rate to rise. As we shall see below, both these mechanisms are crucial for interpreting the impact of domestic sourcing dynamics on inflation.

2.3.3 Aggregate Demand/Supply Interpretation

Collecting the results above, we can define a three equation model that determines the output gap ($\hat{y}_t - \hat{y}_t^n$), consumer price inflation (π_{Ct}), and the interest rate ($\hat{r}_t$). The equilibrium system is given by Equation 34, Equation 35 with the solution for the real natural interest rate (Equation 36) and the definition of the real interest rate ($\hat{\tilde{r}}_t \equiv \hat{r}_t - E_t \pi_{Ct+1}$), and the monetary policy rule $\hat{r}_t = \omega \pi_{Ct}$.

Combining the monetary policy rule with the IS curve, one can define a downward sloping "aggregate demand" (AD) schedule in $\{\pi_{Ct}, \hat{y}_t - \hat{y}_t^n\}$, with π_{Ct} on the y-axis and $\hat{y}_t - \hat{y}_t^n$ on the x-axis, where higher inflation today is associated with lower values of the output gap, since the central bank raises interest rates in response. The Phillips curve is then upward sloping, where a higher value of the output gap today raises current inflation (all else equal). Echoing standard textbooks, this Phillips curve can be thought of as an "aggregate supply" (AS) relation. We emphasize the intuition based on this AD/AS version of the model in describing analytical results below.

2.3.4 Shocks and Inflation in the Three Equation Model

We apply the three equation model to explain the seemingly puzzling inflation results in Section 2.2. Our explanation emphasizes three features of the changes in foreign sourcing – (i) they were long-lasting shocks (with a permanent component), (ii) the shocks were phased in over time, and (iii) the medium term dynamics for domestic input sourcing were u-shaped – there was a pronounced decline in domestic input sourcing followed by a rebound. After using these facts to explain model inflation dynamics, we re-deploy the model to parse the conventional wisdom about the impacts of globalization on inflation.

Interpreting Simulated Inflation Combining points (i) and (ii), the rise in inflation around the year 2000 in the model can be understood as the manifestation of an anticipated future decline in domestic sourcing. Starting from steady state, consider a path for domestic sourcing such that it is unchanged today ($\Delta\hat{\lambda}_{Ht}^C = \Delta\hat{\lambda}_{Ht}^M = 0$), expected to decline in the next period ($\Delta\hat{\lambda}_{Ht+1}^C = \Delta\hat{\lambda}_{Ht+1}^M < 0$), and then expected to stabilize at a permanently lower level in the long run ($\Delta\hat{\lambda}_{Ht+j}^C = \Delta\hat{\lambda}_{Ht+j}^M = 0$ for $j > 1$). Note that this is a stylized three-period depiction of the shocks plotted in Figure 4a, abstracting from medium term dynamics.

This shock has two effects. First, the AD schedule shifts up, due to the rise in the real natural rate implied by the anticipated decline in domestic sourcing. Second, the AS schedule also shifts up, due to the expected decline in domestic sourcing of final goods. Both of these shocks drive inflation up, as in Figure 4b. This is the story of inflationary globalization in the model. Since the phase-in period extends for most of the 1997-2010 period, inflation is persistently above zero as trade rises in the model.

In additional to this phased permanent shock, inflation also responds to medium term dynamics for the sourcing shares. As in point (iii), these dynamics are most pronounced for the input sourcing share, where domestic input sourcing falls rapidly in the first half of the period and then reverts back to its long run level. This rapid decline accentuates the rise in overall inflation in early years. Further, as the shock reverts, the logic inverts: the phased rise in domestic input sourcing after 2010 actually drives inflation down (below zero) in the model.

In addition to this u-shape in input sourcing, there are significant short term effects of changes in domestic sourcing surrounding the Great Trade Collapse in 2008-2010. Anticipation of the collapse (though somewhat implausible), accounts for the decline in inflation in the mid-2000s. More plausibly, anticipated recovery from the trade collapse drives inflation up during Great Recession.

All together, these results are a testament to the importance of trade dynamics (as opposed to levels) in explaining inflation outcomes. Staying with this theme, we pause to discuss the conventional wisdom regarding the impact of rising trade on inflation, and how our argument differs from it.

Parsing Conventional Wisdom Some changes in domestic sourcing do have deflationary impacts. First, immediate declines in domestic final goods sourcing – whether temporary or permanent – lower inflation today. Our view is that the conventional wisdom that increasing trade lowers inflation is largely based on thinking through the impacts of these types of shocks. Second, temporary declines in domestic input sourcing lower inflation. However, immediate permanent declines in domestic input sourcing do not. We discuss these scenarios in sequence now.

Consider first immediate declines in domestic sourcing for final goods, such that $\triangle\hat{\lambda}^C_{Ht} < 0$. If the decline is permanent, then further suppose that $\Delta\hat{\lambda}^C_{Ht+1} = 0$. If it is temporary, assume that sourcing reverts to its original level: $\Delta\hat{\lambda}^C_{Ht+1} = -\Delta\hat{\lambda}^C_{Ht} > 0$. In both these cases, the AS curve – equivalently, the Phillips curve – shifts down on impact, directly lowering inflation. Moreover, the shift is more pronounced for a temporary shock than a permanent shock.[26]

This shift in the Phillips curve captures the standard "relative import prices" intuition for the impacts of trade: a decline in the relative price of imports – i.e., an improvement in Home's terms of trade – reduces consumer price inflation, holding the domestic output gap fixed. This intuition features prominently in central bank policy discussions of the impact of globalization on inflation, where policymakers refer to increases in effective aggregate supply and/or shifting Phillips Curves to explain falling inflation [e.g., Yellen (2006); Bean (2007); Carney (2017)]. Nonetheless, it obviously misses the role of phase-in dynamics in explaining inflation, which are important in both theory and historical experience.

Conventional models and analysis also omit any role for changes in input sourcing, which are quantitatively important in US data. This omission matters, regardless of the nature of the shocks. First, suppose there is an unanticipated immediate, permanent decline in domestic sourcing for consumption goods: $\Delta\hat{\lambda}^M_{Ht} < 0$ and $\Delta\hat{\lambda}^M_{Ht+1} = 0$. In this case, there is no shift in either the AS curve or the AD curve. Thus, the shock has no impact on inflation. This result strikes us as likely to surprise readers steeped in the literature.

Second, suppose that there is temporary decline in domestic sourcing for inputs: $\Delta\hat{\lambda}^M_{Ht} < 0$ and $\Delta\hat{\lambda}^M_{Ht+1} = -\Delta\hat{\lambda}^M_{Ht} > 0$. In this case, the shock has no direct impact on the Phillips Curve (AS curve). Instead, it works through aggregate demand, where the AD curve shifts down, since expected mean reversion in

[26]Examining Equation 34, the shocks enter via the term $\frac{1}{(\eta-1)}\left[\Delta\hat{\lambda}^C_{Ht} - \beta E_t \Delta\hat{\lambda}^C_{Ht+1}\right]$. For a temporary shock, both the decline in domestic sourcing today ($\Delta\hat{\lambda}^C_{Ht} < 0$) and its subsequent rebound ($\Delta\hat{\lambda}^C_{Ht+1} > 0$) reduce inflation today. A temporary shock also leads the AD curve to shift down, since $\Delta\hat{\lambda}^C_{Ht+1} > 0$ lowers the real natural rate and reduces aggregate demand, thus further lowering inflation in period t.

the shock ($\Delta\hat{\lambda}^M_{Ht+1} > 0$) lowers the real natural rate. While rising trade induces a decline in inflation in this scenario – consistent with conventional wisdom – the mechanism is entirely implausible: inflation falls because the transitory shock induces a recession (negative output gap) in period t. While much policy discussion emphasizes the possible deflationary impact of globalization, we know of no prominent discussion that emphasizes its recessionary impacts to explain the decline in inflation.

To conclude, this discussion re-emphasizes two of our main points. Trade dynamics drive inflation, and this point is largely overlooked in prior work on globalization and inflation. Furthermore, how trade dynamics matter depends on whether trade in final goods or inputs is changing.

3 Extensions to the Baseline Model: Financial Inflow Shocks and Import Competition

We now consider two substantive extensions to the baseline model.[27] The first extension is to introduce financial inflow shocks that allow the model to match the path of the US trade deficit over time. The second extension is to introduce variable markups and dollar currency pricing in the model, allowing for import competition to influence prices. For clarity sake, we examine these modifications one at a time.

3.1 Financial Inflow Shocks and Trade Imbalances

In addition to trade integration, globalization was also associated with increased financial integration and a US current account (trade) deficit. In the baseline model, we assumed that international financial markets were complete, so current account imbalances were determined by consumption risk sharing. Further, we omitted shocks – to exogenous foreign variables or financial integration itself – that target the actual path of trade and current account deficits over time. As a result, the trade deficit in the baseline model differs from data.

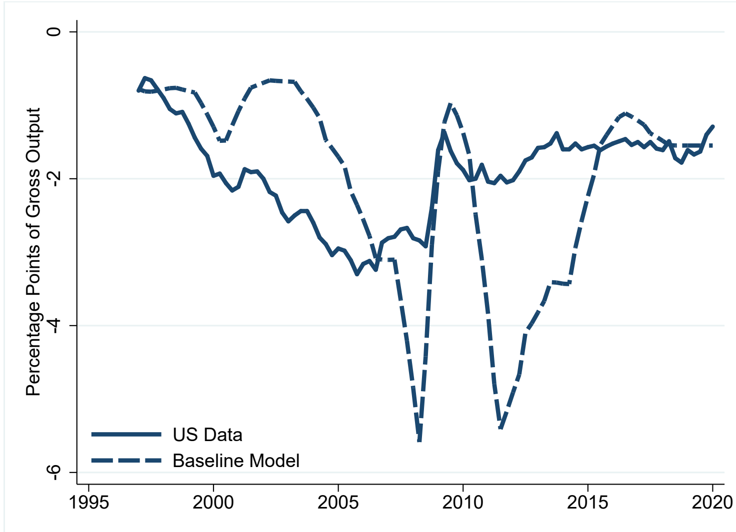
We plot the trade deficit in the baseline model (with shocks to domestic sourcing) and official US quarterly data in Figure 5, both expressed as a share of gross output. While the baseline model generates a widening of the US trade deficit through the mid-2000s, it under predicts the deficit during this "global savings glut" period. Further, the model yields large swings in the deficit around the Great Recession, and fails to pick up on the sustained improvement in the trade deficit during the 2010-2015 interval.

Given these discrepancies, one naturally wonders whether deviations from perfect risk sharing, or foreign shocks that influence international financial flows, which are needed to match the actual evolution of US trade imbalances, are important for our inflation results? Specifically, we emphasized that trade integration drives up the real natural rate of interest in the three equation model, which stokes inflation. A common view is that the global savings glut drove down real interest rates during this period, so one might think this would lower inflation as a result. To investigate this mechanism, we develop an extension to baseline model that allows us to study the role of the trade balance in driving inflation.

To incorporate financial inflow shocks in the model, we drop the complete markets assumption, and replace it with a simple alternative framework that takes the path of the trade deficit as given. Using notation from previous sections, the trade deficit in the model is $TD_t = P_{Ft}\tau_{Ct}C_{Ft} + P_{Ft}\tau_{Mt}M_{Ft} - P_{Ht}X_t$. Now define $TDY_t \equiv \frac{TD_t}{P_{Ht}Y_t}$ to be the ratio of the trade deficit to nominal gross output. Taking the ratio of

[27] We additionally consider a third extension to the baseline model in Appendix C, where we introduce physical capital into the model, in line with medium-scale New Keynesian models commonly used for monetary policy analysis. The punchline is that the evolution of inflation in the baseline model is robust to introducing physical capital.

Figure 5: Trade Deficit (% GDP)



Note: The line labeled US Data is the ratio of the quarterly US trade balance (from the BEA) to gross output. We measure quarterly gross output by dividing quarterly GDP data from the BEA by the ratio of value added to gross output (measured at an annual frequency) in BEA input-output data.

the trade balance to gross output (TDY_t) as given, we can then replace the risk-sharing condition in the original model with this statement of the (exogenous) trade balance. With slight abuse of language, we refer to TDY_t as a "financial inflow shock" in discussion that follows.

This approach gives us an additional degree of freedom to match the evolution of the US trade deficit. Further, the presence of this shock in the model may also have an independent impact on inflation. To analyze these effects, we turn to the three equation model, augmented to include the financial inflow shock.

3.1.1 Financial Inflow Shocks in the Three Equation Model

In Appendix D, we provide a full treatment of the three equation model with financial inflow shocks. We show first that these shocks do not enter the Phillips curve directly. We then re-derive the IS curve, where financial inflow shocks play an important role. Superficially, the IS curve is still given by Equation 35. However, under the hood, financial inflow shocks influence the real natural rate of interest, which is given by:

$$\hat{\tilde{r}}_t^n = -\tilde{\Upsilon}_M E_t\left(\Delta\hat{\lambda}^M_{Ht+1}\right) - \tilde{\Upsilon}_C E_t\left(\Delta\hat{\lambda}^C_{Ht+1}\right) + \tilde{\Upsilon}_{TD} E_t\left(\Delta\widehat{tdy}_{t+1}\right) \tag{37}$$

where $\tilde{\Upsilon}_M > 0$, $\tilde{\Upsilon}_C > 0$ if and only if $\rho > 1$, and $\tilde{\Upsilon}_{TD} > 0$ (see the appendix for exact formulas).[28]

In Equation 37, note that the dynamics of the trade deficit across periods determine the natural interest rate, not the level of the trade deficit itself. If the trade deficit is expected to widen from t to $t+1$, such that $\Delta\widehat{tdy}_{t+1} > 0$, then the natural rate of interest rises. The reason is that an expected increase in the trade deficit is associated with higher future consumption relative to present consumption, and this demands a higher interest rate via the Euler equation.

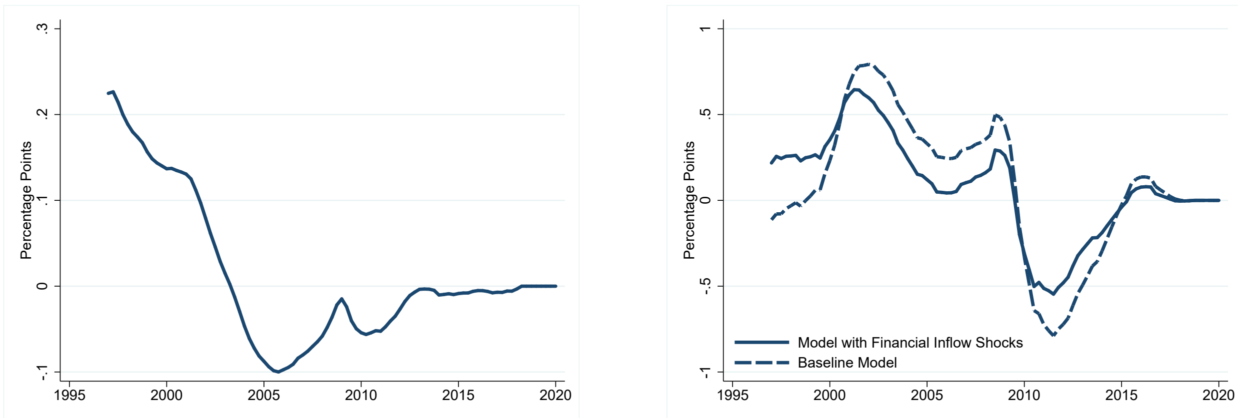
The dependence of the natural rate on changes in capital inflows, rather than their level, runs counter to the intuition we discussed above about how one might think that the global savings glut and resulting US trade deficit should lower the real interest rate. It is the case that a one-time positive shock to capital

[28]To simplify this expression, we suppress shocks to foreign consumption and productivity.

Figure 6: Simulation of Inflation in the Model with Financial Inflow Shocks

(a) Simulated Inflation with Financial Inflow Shocks

(b) Simulated Inflation with Financial Inflow and Domestic Sourcing Shocks



inflows lowers the natural interest rate. In this case, $\widehat{tdy}_t > 0$ and $E_t\widehat{tdy}_{t+1} = 0$, so that $E_t\Delta\widehat{tdy}_{t+1} < 0$. This temporary capital inflow shock would lead domestic agents to consume more in period t, which lowers the natural rate of interest (via the the Euler equation) and ultimately inflation. However, this is not the kind of shock that is relevant in the early 2000's.

Referring back to Figure 5, the early 2000's saw sustained increases in the trade deficit year over year. Thus, their effects do not conform to this one-off shock intuition. The phased widening of the deficit led to a sequence of years in which $\Delta\widehat{tdy}_{t+1} > 0$. Thus, anticipation of widening deficits over this period exerted upward pressure on the natural interest rate, rather than downward pressure. Again, this stokes inflation. In contrast, closure of the deficit from the mid-2000s onward exerts downward pressure on the natural rate of interest and thus inflation. We will see these dynamics borne out in the historical simulations that incorporate this shock.

3.1.2 Inflation Dynamics with Financial Inflow Shocks

Following the same procedure for simulating the model as in previous sections, we plot the evolution of inflation given the path of observed trade deficits ($\widehat{tdy}_t$) in Figure 6a, with domestic sourcing shocks set to zero. Consistent with the narrative above, the widening of the US trade deficit actually serves to push up inflation in early years, and then the anticipated closure of it yields disinflation in the middle years. Note also the magnitudes in this figure: the capital inflow shocks alone yield modest inflation/deflation, less pronounced than the impact of the domestic sourcing shocks.

In Figure 6b, we plot simulated inflation with all three shocks active (the capital inflow shock plus the two domestic sourcing shocks) in this model. For comparison, we also plot inflation from simulation of the baseline model. Consistent with the muted impacts of the capital inflow shocks, the qualitative and quantitative results we obtained in the baseline continue to hold in this extended model. Therefore, we conclude that inflation dynamics do not seem to have been impacted much by current account dynamics during this time frame, above and beyond the impact of changes in domestic sourcing. Further, since this model drops the complete markets assumption, we also conclude that financial market structure does not play a significant role in our results.

3.2 Import Competition with Variable Markups

Thus far, we have considered models with CES preferences and production functions. While these models feature variable markups due to price adjustment frictions, they do not allow optimal (flexible price) markups to vary with import competition. This omits any potential role for import competition in lowering markups, and thus influencing inflation. To incorporate pro-competitive effects of trade, we extend the model to incorporate Kimball (1995) style final goods and input aggregators. As in Gopinath et al. (2020), we combine Kimball aggregation with dominant (dollar) currency pricing of imports. One distinct contribution of this section will be to demonstrate that our sufficient statistics approach to analyzing the model can be applied in this more sophisticated setting.

3.2.1 Main New Assumptions and Results

There are three important changes in this version of the model relative to the baseline. First, we assume now that aggregators for final and intermediate goods are given by:

$$\nu \int_0^1 \Upsilon\left(\frac{C_{Ht}(i)}{\nu C_t}\right) di + (1-\nu)\int_0^1 \Upsilon\left(\frac{C_{Ft}(i)}{(1-\nu)C_t}\right) di = 1 \tag{38}$$

$$\xi \int_0^1 \Upsilon\left(\frac{M_{Ht}(i)}{\xi M_t}\right) di + (1-\xi)\int_0^1 \Upsilon\left(\frac{M_{Ft}(i)}{(1-\xi)M_t}\right) di = 1, \tag{39}$$

where $C_{Ft}(i)$ and $M_{Ft}(i)$ are consumption of individual foreign varieties, the parameters ν and ξ govern home bias, and the function $\Upsilon(\cdot)$ satisfies $\Upsilon(1) = 1$, $\Upsilon^{'}(\cdot) > 0$, and $\Upsilon^{''}(\cdot) < 0$. As in Klenow and Willis (2016) and Gopinath et al. (2020), we parameterize $\Upsilon(\cdot)$ using a flexible functional form:

$$\Upsilon(x) = 1 + (\sigma - 1)\exp\left(\frac{1}{\varepsilon}\right)\varepsilon^{\sigma/\varepsilon - 1}\left(\Gamma\left(\frac{\sigma}{\varepsilon}, \frac{1}{\varepsilon}\right) - \Gamma\left(\frac{\sigma}{\varepsilon}, \frac{x^{\epsilon/\sigma}}{\varepsilon}\right)\right), \tag{40}$$

where $\Gamma(u, z) = \int_z^\infty s^{u-1}e^{-s}ds$ is the incomplete gamma function, with $\sigma > 1$ and $\varepsilon > 0$. Given these aggregators, import demand for final goods and inputs now features variable elasticities, so optimal markups for Home firms vary with aggregate market conditions – in particular, they are lower when import competition is tough.

Second, we assume that import prices are set in dollars, subject to Rotemberg adjustment costs. The solution to the pricing problems for foreign firms yields a dynamic pricing equation for imports, similar to the price setting equations for domestic firms.

Third, we assume that Home and Foreign markets are segmented, so Home producers set prices independently for domestic and export sales. This simplifies the dynamics of the the domestic price level, but plays an otherwise minor role in the results.

We provide a full characterization of the model in Appendix E. Here we emphasize three key results for understanding how we use this model and interpret the results.

The first result is that the sufficient statistics approach to model analysis continues to apply in this model. To sketch the argument, the log linear approximation to the first order condition for demand for domestic final goods in the symmetric firm equilibrium is:

$$\hat{c}_{Ht} = -\sigma\left(\frac{C_{H0}}{\nu C_0}\right)^{-\varepsilon/\sigma}\left(\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct}\right) + \hat{c}_t \tag{41}$$

where $\hat{d}_{Ct}$ is an endogenous term that indexes the level of demand under Kimball aggregation, and the subscript 0 denotes steady state values. This result implies that the log-linear approximation of demand has a constant elasticity, governed by parameters and steady state values. In the calibrated steady state, $C_{H0} = \nu C_0$. Further, we show in the appendix that $\hat{d}_{Ct} = 0$ in the solution to the log-linearized model. Combining these two observations, we can then solve for the relative price of home goods, just like in the baseline model: $\hat{p}_{Ht} - \hat{p}_{Ct} = -\frac{1}{\sigma-1}\hat{\lambda}^C_{Ht}$. Analogously, $\hat{p}_{Ht} - \hat{p}_{Mt} = -\frac{1}{\sigma-1}\hat{\lambda}^M_{Ht}$ for imported inputs. Thus, we can substitute for these relative prices throughout the model to write the equilibrium in terms of deviations in domestic sourcing shares from the steady state. Taking values for $\hat{\lambda}^C_{Ht}$ and $\hat{\lambda}^M_{Ht}$ as given, these results imply that we need not solve for the relative price of domestic final goods or inputs.[29]

The second result concerns the domestic price Phillips curve, which reflects optimal pricing for Home firms. In the log-linear symmetric firm equilibrium, it is given by:

$$\pi_{Ht} = \frac{-\hat{\epsilon}_{Ht}}{\phi} + \frac{(\epsilon_{H0} - 1)}{\phi}\widehat{rmc}_t + \beta E_t\left(\pi_{Ht+1}\right), \tag{42}$$

where $\widehat{rmc}_t = \hat{mc}_t - \hat{p}_{Ht}$. Here ϵ_{H0} is the elasticity of demand faced by Home firms for sales to domestic buyers in steady state, and $\hat{\epsilon}_{Ht}$ is the log deviation in this elasticity of demand at date t from its steady state value. When the elasticity of demand is larger than its steady state value ($\hat{\epsilon}_{Ht} > 0$), then Home firms reduce their markups and thus domestic price inflation is lower (all else equal). For further insight, we can characterize this demand elasticity as function of primitive domestic sourcing shares in the model (see Appendix E for derivation):

$$\hat{\epsilon}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{\epsilon}^C_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{\epsilon}^M_{Ht} \tag{43}$$

$$\text{with} \quad \hat{\epsilon}^C_{Ht} = -\left(\frac{\varepsilon}{\sigma-1}\right)\hat{\lambda}^C_{Ht} \tag{44}$$

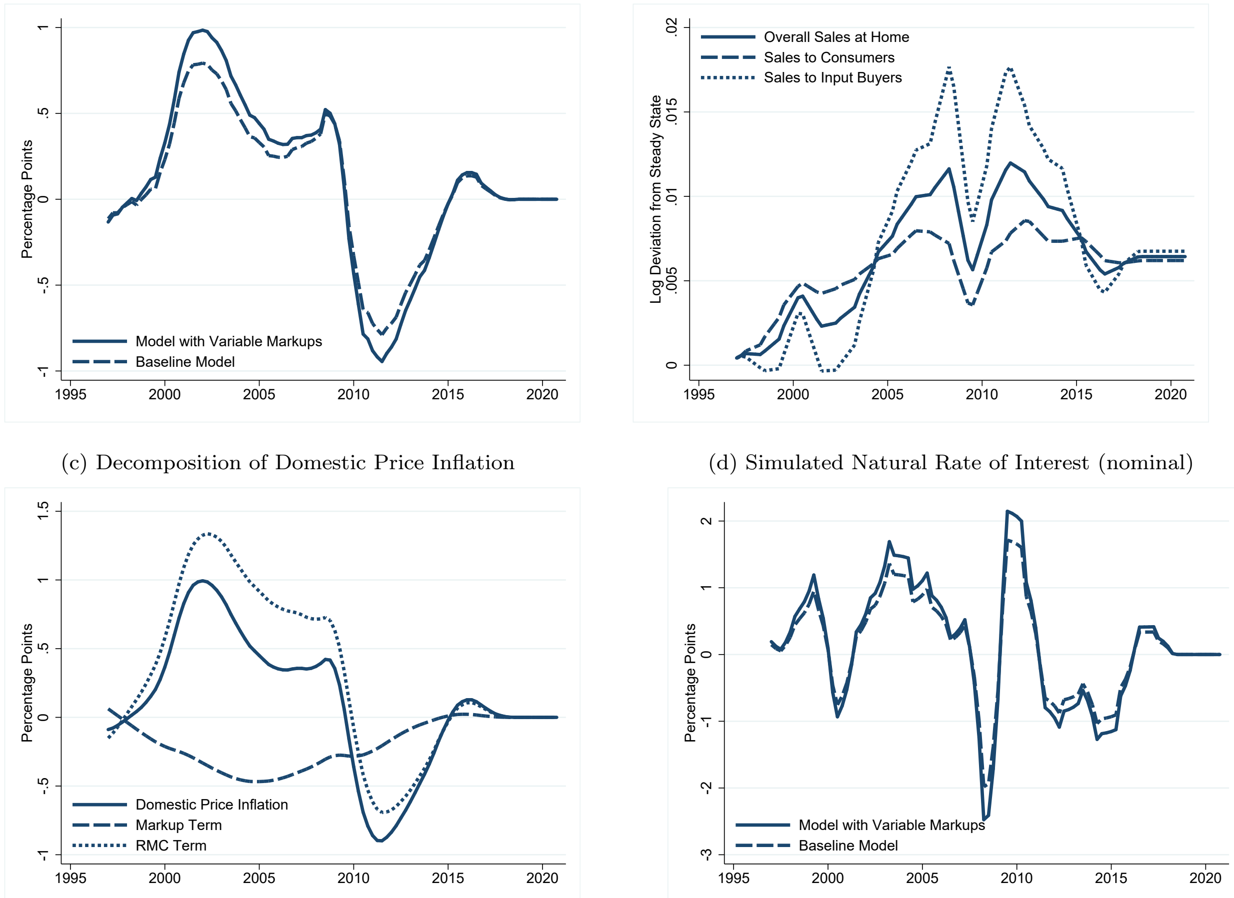
$$\text{and} \quad \hat{\epsilon}^M_{Ht} = -\left(\frac{\varepsilon}{\sigma-1}\right)\hat{\lambda}^M_{Ht}. \tag{45}$$

Thus, the elasticities by end use $\hat{\epsilon}^C_{Ht}$ and $\hat{\epsilon}^M_{Ht}$ are decreasing functions of domestic sourcing, where the parameter ε controls the elasticity of markups to relative prices. This is the import competition channel: demand elasticities are lower (markups are higher) when domestic sourcing is high. For readers familiar with markup shocks in the New Keynesian literature, import competition restrains markups, operating like a markup shock in this Phillips curve.

While this second result is one channel through which variable markups influence inflation, there is a distinct role for markups operating via the supply side of the model. The third key result from the model is that the evolution of markups over time influences the natural interest rate, via their effects on flexible price consumption dynamics. As in standard monopolistic competition models, markups depress equilibrium output – they deter input use and distort labor supply down by lowering real wages. An expected decline in domestic sourcing reduces expected future markups for Home firms. In turn, it raises expected output in the future relative to current output, which generates an expected real exchange rate depreciation. Via risk sharing, the expected depreciation raises the expected growth rate of consumption, and thus the natural interest rate today. We will show via simulation below that this third channel, via which the path of markups

[29]This implies that we need not actually solve the import pricing problem to characterize equilibrium variables that determine domestic inflation. To make this argument transparent, we first write down the full equilibrium that includes the solution for the import pricing problem in Appendix E, and then we reduce the model down using the sufficient statistics argument.

Figure 7: Simulation of Inflation in the Model with Variable Markups



(a) Simulated Consumer Price Inflation

(b) Simulated Demand Elasticity for Home Goods

(c) Decomposition of Domestic Price Inflation

(d) Simulated Natural Rate of Interest (nominal)

influences the level of aggregate demand – is important for understanding inflation dynamics.

3.2.2 Inflation Dynamics with Variable Markups

With these key results in hand, we proceed directly to simulation. We calibrate the model with variable markups to match the same initial steady state as the baseline model. Further, we set the parameter $\sigma = 3$ in the Klenow-Willis Υ-function, which controls elasticity of substitution between home and foreign goods in the steady state, to match the baseline model. Following Gopinath et al. (2020), we set $\varepsilon = 1$, which governs the elasticity of the elasticity of substitution with respect to relative prices.

In Figure 7a, we plot inflation in this variable markups model, along with results from the baseline model. The model with variable markups features a larger increase in inflation during the 2000-2010 period than the baseline model, and then a stronger disinflation during the post-2010 period. Far from dampening the impact of rising trade on inflation, variable markups actually make inflation more responsive to changes in trade. Cumulative inflation from 1997 to 2018 is also higher in the model with variable markups, leaving the price level two percentage points higher than in the baseline simulation ($\approx 10\%$ versus $\approx 8\%$ total increase in consumer prices).

To inspect the mechanism, we plot the (log-deviation from the steady state of the) elasticity of demand for Home firms ($\hat{\epsilon}_{Ht}$) over time in Figure 7b, along with the underlying elasticities of demand for Home final goods ($\hat{\epsilon}^{C}_{Ht}$) and inputs ($\hat{\epsilon}^{M}_{Ht}$) separately. Per the discussion above, note these track the inverted path of changes in domestic sourcing over time, so that the elasticity of demand peaks (markups are lowest) in the mid-2000s. These elasticities determine the size of the effective "markup shock" in the domestic price Phillips curve (Equation 42).

To illustrate the influence of these changes in markups on inflation at each point in time, we solve the domestic price Phillips curve forward. However, because markups change in perpetuity due to the permanent nature of the shocks, we adjust the standard inflation accounting equation to account for this feature of our exercise. Denoting the terminal steady state by T, the following relationship between markups and real marginal costs holds: $\widehat{rmc}_T = -\left(\frac{1}{\epsilon_{H0}-1}\right)\hat{\epsilon}_{HT}$, and domestic price inflation in the terminal steady state is consequently zero. If the economy jumped immediately to this long run equilibrium, then inflation would be given by $\tilde{\pi}_H = -\frac{1}{\phi}\sum_{s=0}^{\infty}\beta^s\hat{\epsilon}_{HT} + \left(\frac{\epsilon_{H0}-1}{\phi}\right)\sum_{s=0}^{\infty}\beta^s\widehat{rmc}_T = 0$. We look at deviations in actual date-t inflation from this hypothetical benchmark:

$$\pi_{Ht} = \underbrace{-\frac{1}{\phi}\sum_{s=0}^{\infty}\beta^s E_t\left[\hat{\epsilon}_{Ht+s} - \hat{\epsilon}_{HT}\right]}_{\text{Markup Term}} + \underbrace{\left(\frac{\epsilon_{H0}-1}{\phi}\right)\sum_{s=0}^{\infty}\beta^s E_t\left[\widehat{rmc}_{t+s} - \widehat{rmc}_T\right]}_{\text{RMC Term}}. \tag{46}$$

We plot the results of this decomposition in Figure 7c. The markup term evidently depresses inflation, by almost 50 basis points when its impact is largest in the mid-2000s when domestic sourcing is high. Nonetheless, this pro-competitive effect is more than offset by endogenous changes in real marginal costs, which drives the increase in domestic price inflation overall. Further, pro-competitive effects play no role in explaining the pivot from inflation to deflation after 2010.

To understand why real marginal costs more than offset the direct impacts of markup restraint on inflation, we turn a discussion of the natural interest rate. As noted above, markup shocks not only shift the Phillips Curve, they also influence the level of aggregate demand. Aggregate demand rises as markups decline, through the impact of time-variation in markups on the natural interest rate. In Figure 7d, we show that the (nominal) natural interest rate in this variable markups model is more pro-cyclical than in the baseline model with constant markups. As we asserted, the natural rate rises in anticipation of declines in domestic sourcing, and more forcefully in the variable markups model than in the baseline. Again, this is due to the fact that reductions in markups expand output, via their impacts on labor supply and intermediate input use.

To sum up, like the capital inflow shocks above, allowing for variable markups in the model does not substantially impact our overall conclusions regarding the impact of trade dynamics on inflation. This is surprising, in that pro-competitive effects of trade on domestic prices are commonly cited as an important element in studies of price changes across sectors in response to trade [Auer and Fischer (2010); Jaravel and Sager (2019); Bai and Stumpner (2019)]. We have shown that pro-competitive effects are not important in a fully specified general equilibrium context. Inflation dynamics are largely unrelated to markup dynamics. Moreover, reductions in markups actually raise inflation relative to the baseline model with constant markups, as their impacts on aggregate demand via the natural interest rate more than offset their impact through the Phillips curve.

4 Multisector Model

We now turn to a multisector version of the baseline model. As we discussed in Section 1, domestic sourcing shares have changed in heterogeneous ways across sectors, and we apply the multisector model to study the implications of this heterogeneity for inflation. In doing so, we also come full circle to tie our results back to the multisector accounting framework we introduced in Section 1.

4.1 Model Overview

The multisector model, with sectors indexed by $s \in \{1, \ldots, S\}$, is an extension of the baseline model.[30] We describe rudiments of the model here, and present the full model in the Appendix F.

On the consumption side, we adopt a standard nested CES framework, where aggregate consumption is given by:

$$C_t = \left(\sum_s \zeta(s)^{1/\vartheta} C_t(s)^{(\vartheta-1)/\vartheta} \right)^{\vartheta/(\vartheta-1)} \tag{47}$$

$$C_t(s) = \left(\nu(s)^{1/\eta(s)} C_{Ht}(s)^{(\eta(s)-1)/\eta_C(s)} + (1-\nu(s))^{1/\eta(s)} \, C_{Ft}(s)^{(\eta(s)-1)/\eta(s)} \right)^{\eta(s)/(\eta(s)-1)} \tag{48}$$

$$C_{Ht}(s) = \left(\int_0^1 C_{Ht}(s,i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}, \tag{49}$$

where $C_t(s)$ is aggregate consumption of the sector-s composite goods, which is a CES composite of domestic $(C_{Ht}(s))$ and foreign $(C_{Ft}(s))$ final goods. In the system, $\vartheta \in [0, \infty)$ is the elasticity of substitution across sector composites, and $\eta(s) \in [0, \infty)$ is the sector-specific elasticity of substitution between home and foreign composites. The CES weights satisfy $\sum_s \zeta(s) = 1$ and $\nu(s) \in [0, 1]$.

On the production side, individual varieties (indexed by i) are produced by combining labor with inputs, where inputs take on a nested CES structure:

$$Y_t(s,i) = Z_t(s) \left(L_t(s,i) \right)^{1-\alpha(s)} \left(M_t(s,i) \right)^{\alpha(s)} \tag{50}$$

$$M_t(s,i) = \left(\sum_{s'} \left(\alpha(s',s)/\alpha(s) \right)^{1/\kappa} M_t(s',s,i)^{(\kappa-1)/\kappa} \right)^{\kappa/(\kappa-1)} \tag{51}$$

$$M_t(s',s,i) = \left[\xi(s',s)^{\frac{1}{\eta(s')}} M_{Ht}(s',s,i)^{\frac{\eta(s')-1}{\eta(s')}} + (1-\xi(s',s))^{\frac{1}{\eta(s')}} M_{Ft}(s',s,i)^{\frac{\eta(s')-1}{\eta(s')}} \right]^{\frac{\eta(s')}{\eta(s')-1}} \tag{52}$$

where $M_{Ht}(s',s,i)$ is the quantity of a composite home good from sector s' used by firm i in sector s, $M_t(s',s,i)$ is the composite input from sector s' used by firm i in sector s, which aggregates $M_{Ht}(s',s,i)$ and a composite foreign input $M_{Ft}(s',s,i)$, and $M_t(s,i)$ is the overall composite input used by firm i in sector s. The parameter $\kappa \in [0, \infty)$ is the elasticity of substitution across sectors in input use, and parameter restrictions $\alpha(s) \in [0, 1)$, $\sum_{s'} \alpha(s',s) = \alpha(s)$, and $\xi(s',s) \in [0, 1]$ hold.

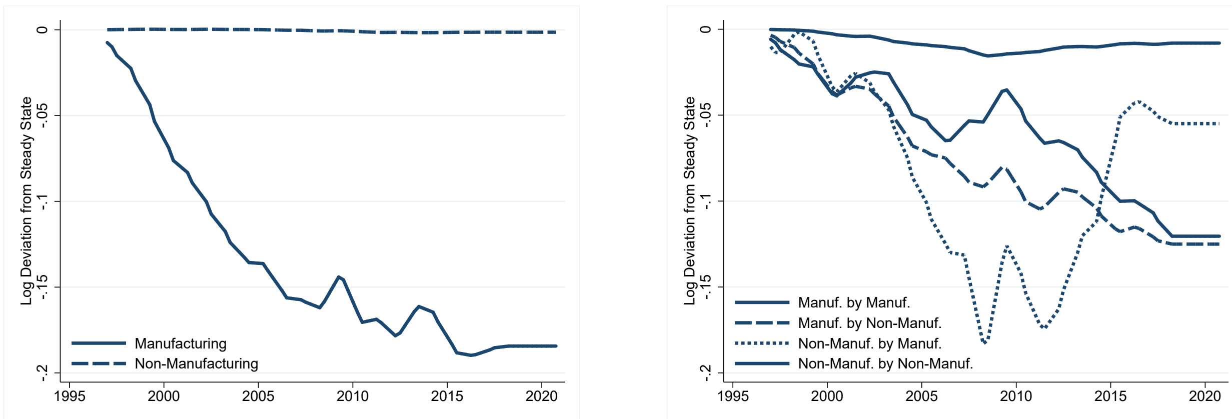
As in the baseline model, we can again use domestic sourcing shares as sufficient statistics. Due to the multisector structure, there are now $S + S^2$ domestic sourcing shares $\left\{ \lambda_{Ct}(s), \lambda_{Mt}(s',s) \right\}$ that are proxies

[30] For analytical clarity, we revert back to CES production functions and preferences, and we assume financial markets are complete, as in the baseline model. The model could easily be extended to include the refinements in Section 3, without altering the main results.

Figure 8: Changes in Domestic Sourcing by Sector

(a) Domestic Sourcing for Final Goods: $\hat{\lambda}^C_{Ht}(s)$

(b) Domestic Sourcing for Inputs: $\hat{\lambda}^M_{Ht}(s', s)$



for relative prices $\left\{\frac{P_{Ht}(s)}{P_{Ct}(s)}, \frac{P_{Ht}(s')}{P_{Mt}(s',s)}\right\}$, where $P_{Ht}(s)$ is the price of a representative producer in sector s, $P_{Ct}(s)$ is the price of the composite final good for sector s, and $P_{Mt}(s^{'}, s)$ is the price of the composite input purchased by sector s from sector $s^{'}$.

4.2 Inflation Dynamics with Two Sectors

We apply the same procedure to simulate the multisector model, as in previous sections. Here we present results for a two sector version of the model, distinguishing manufacturing from a composite non-manufacturing sector that includes agriculture, natural resources, and services. New parameters that govern sector expenditure shares, value-added to output ratios, and the input-output structure across sectors are set to match US input-output data in the initial period (parameter values are recorded in Appendix F). Throughout the simulations that follow, we impose Cobb-Douglas preferences across sectors in final consumption and input use, equivalent to setting $\kappa = 1$ and $\vartheta = 1$. In most simulations (with one exception discussed below), we set $\eta(s) = \eta = 3$, similar to the baseline model.

Changes in domestic sourcing are plotted in Figure 8. In Figure 8a, changes in final goods sourcing ($\hat{\lambda}^C_{Ht}(s)$) are completely different across sectors – domestic sourcing falls by almost 20 percent in manufacturing, while it is nearly unchanged in non-manufacturing. In Figure 8b, we plot changes in domestic sourcing for inputs ($\hat{\lambda}^M_{Ht}(s', s)$). Again, there is a pronounced decline in domestic sourcing of manufacturing inputs by both the manufacturing and non-manufacturing sectors, of about 12 percent each. There are very heterogeneous developments for sourcing of non-manufacturing inputs across sectors, however. There is minimal change in domestic sourcing of non-manufacturing inputs by the non-manufacturing sector itself. In contrast, there is initially a large decline in domestic sourcing of non-manufacturing inputs by the manufacturing sector, which then reverses after 2010. We explore the role of this particular shock in several different ways below.

Feeding these changes in domestic sourcing through the model, we plot the resulting consumer price inflation series in Figure 10a. Similar to the baseline model, inflation is positive throughout most of the two decades of rising trade, with the largest positive inflation rates realized in the 2000-2010 period. Thus, dis-aggregating the model to allow for multiple sectors does not qualitatively change the main conclusions we drew from analysis of the one sector model. Further, quantitative magnitudes are quite similar as well.

In Figure 10b, we examine simulated inflation for final goods and input shocks fed into the model separately. Similar to the baseline model, input sourcing shocks dominate the medium term dynamics for inflation. Changes in domestic sourcing for final goods drive inflation up in the first few years of the simulation in particular, and they never drive inflation much below zero.

In Figure 10c, we disaggregate the shocks by sector, rather than by end use, and plot simulated data for manufacturing and non-manufacturing shocks separately. One point that stands out here is that the dynamics of sourcing in the non-manufacturing sector are important in explaining the dip in inflation in the post-2010 period, while both shocks drive inflation up in the pre-2010 period.

Another way to emphasize this result is to examine an alternative simulation with unequal elasticities across sectors. We consider a scenario in which the elasticity between home and foreign goods is higher for non-manufacturing than manufacturing, with an elasticity of 2 in manufacturing and an elasticity of 6 in non-manufacturing. This is a plausible case, since non-manufacturing includes agriculture and natural resources. In Figure 10d, we plot the results from this heterogeneous elasticity simulation, along with the equal elasticity simulation (repeated from Figure 10a). We note that inflation is substantially higher in the 2010-2020 period in this simulation. The reason is that larger elasticity for the non-manufacturing sectors shrinks the importance of the u-shaped evolution of domestic sourcing of non-manufacturing inputs by the manufacturing sector, because it reduces the implied change in the relative price of home versus foreign non-manufactured inputs. In general, however, allowing heterogeneous elasticities does not change the overall behavior of the inflation.

4.3 Accounting for Price Changes in the Multisector Model

In Section 1, we presented introductory results regarding relative prices and price level accounting that suggested rising trade plays a large role in explaining price changes over time. In contrast, throughout the model simulations, we have emphasized the exact opposite result: inflation rises due to increasing trade. As a final exercise, we bring these results together, by demonstrating that the basic accounting results hold in this multisector model.

Using the equilibrium equations presented in Appendix F, we can write sector-level inflation in domestic goods prices as:

$$\pi_{Ht} = \left[\mathbf{I} - \mathbf{A}'\right]^{-1} \left[\mathbf{I} - \alpha\right] \pi_{Vt} + \left(\frac{1}{\eta - 1}\right) \underbrace{\left[\mathbf{I} - \mathbf{A}'\right]^{-1} \left[\mathbf{A}' \circ \Delta\hat{\lambda}'_{Ht}\right] \iota}_{\text{Offshoring Shock}}, \tag{53}$$

where π_{Ht} is a $S \times 1$ vector with elements $\pi_{Ht}(s)$, π_{Vt} is a vector of sector-specific inflation rates for the price of real value added (i.e., sectoral GDP deflators), $\mathbf{A}$ is the steady state input-output matrix, α is a diagonal matrix of steady state shares of inputs in gross output, and $\hat{\lambda}_{Ht}$ is a matrix with elements $\hat{\lambda}(s', s)$ where s' indexes row and s indexes column.[31]

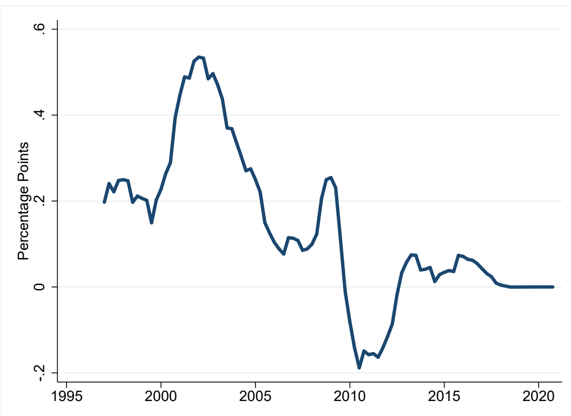
Accumulating these sector-level inflation rates and the offshoring shock over time, we plot the evolution of the simulated relative price of manufacturing goods ($P_{Ht}(m)/P_{Ht}(n)$) over time, along with the cumulative impact of the Offshoring Shock term, in Figure 10a. This figure is analogous to Figure 3b.[32] As in the accounting exercise, we see that the relatively large size of the offshoring shock for manufacturing industries

[31] We present the formula here imposing $\eta(s) = \eta$. This facilitates comparison to Equation 12, and it matches our main parameterization of this model.

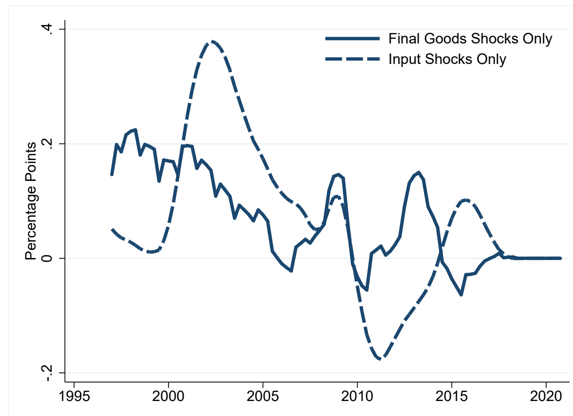
[32] To be clear, the data series plotted in Figure 3b are not strictly comparable to the simulated data and aggregated shocks in the model. In Figure 3b, we aggregated by taking simple means of highly disaggregated manufacturing and non-manufacturing sectors, while here we are plotting simulation output from the two-sector model.

Figure 9: Simulated Inflation in the Multisector Model

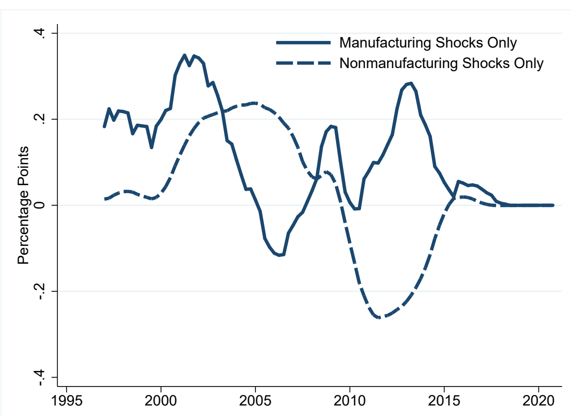
(a) Simulated Inflation in the Multisector Model



(b) Final Goods vs. Input Shocks



(c) Manufacturing vs. Non-Manufacturing Shocks



(d) Heterogeneous Trade Elasticity Across Sectors

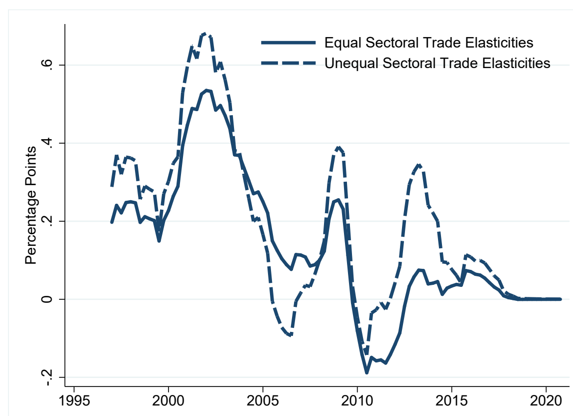
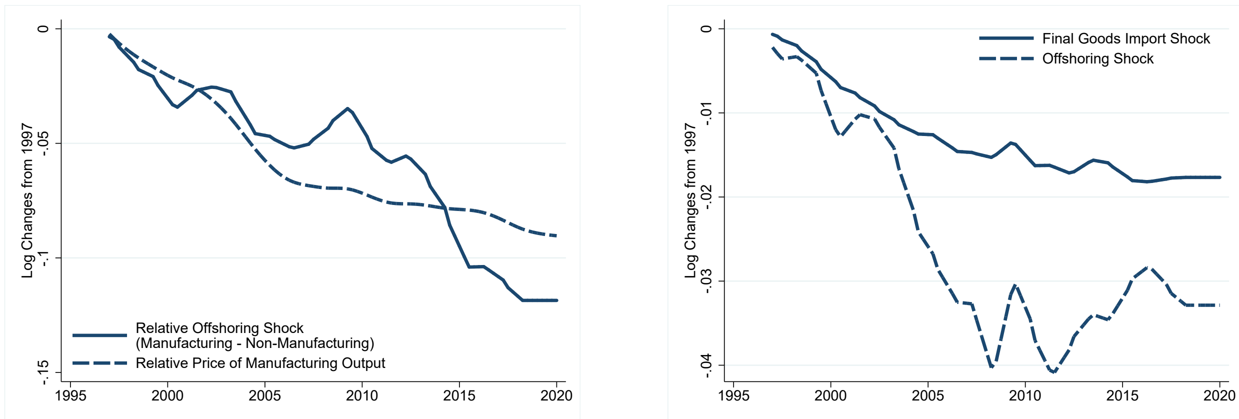


Figure 10: Accounting for Price Changes in the Multisector Model

(a) Relative Price and Offshoring Shock for Manufacturing vs. Non-Manufacturing

(b) Accounting for Changes in the Price Level



Note: In Panel (a), the relative price of manufacturing output is equal to $\sum_{s=1997}^{t} D\pi_{Ht}$ and the relative offshoring shock is $\sum_{s=1997}^{t} D\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{A}'\circ\Delta\hat{\lambda}'_{Hs}\right]\iota$, where D is a 1×2 matrix with elements 1 and -1 that takes differences across sectors. In Panel (b), the final goods import shock term is $\sum_{s=1997}^{t}\left(\frac{1}{\eta-1}\right)\gamma\Delta\hat{\lambda}_{Hs}^{C}$ and the offshoring shock term is $\sum_{s=1997}^{t}\left(\frac{1}{\eta-1}\right)\gamma\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{A}'\circ\Delta\hat{\lambda}'_{Hs}\right]\iota$.

drives down the relative price of manufacturing in the model, by about twelve percent in the long run. This is a causal statement in the model, where all else is held constant, unlike in the accounting exercise. Note too that the relative price diverges from relative offshoring – both at any given time and in the long run, due to the internal dynamics of π_{Vt} in the model in response to shocks.

Turning to price level accounting, we can write aggregate inflation in the model as:

$$\pi_t = \gamma\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{I}-\alpha\right]\pi_{Vt} + \underbrace{\left(\frac{1}{\eta-1}\right)\gamma\left[\mathbf{I}-\mathbf{A}'\right]^{-1}\left[\mathbf{A}'\circ\Delta\hat{\lambda}'_{Ht}\right]\iota}_{\text{Offshoring}} + \underbrace{\left(\frac{1}{\eta-1}\right)\gamma\Delta\hat{\lambda}_{Ht}^{C}}_{\text{Final Goods Imports}}, \tag{54}$$

where γ is a row vector of sector-level consumption shares. This parallels Equation 17 in Section 1.[33] In Figure 10b, we plot the impact of changes in offshoring and final goods sourcing on the consumer price level, adding up quarter-on-quarter changes in the model. Like in the accounting exercise, the cumulative impact of offshoring and increased foreign sourcing of final goods appears to be important in restraining the aggregate price level, accounting for reductions in the price level of 5-6%, or 25-30 basis points per year.

Having developed the model counterfactuals in full, we are now in a position to emphasize again that these accounting results tell us almost nothing about the actual role of trade in restraining inflation. As we have shown, rising trade pushes inflation up. The accounting exercises mislead precisely because a rise in offshoring and final goods imports triggers increases in the price of domestic real value added that more than offset the improvement in the terms of trade. This speaks to the importance of well-defined counterfactuals in tying trade to inflation. Prior work that has demonstrated that trade restrains price growth across sectors, while useful for many purposes, is not a good guide to how trade impacts inflation per se.

[33] We are careful here to call the final term "Final Goods Imports" rather than "C Imports," because final consumption in the model includes consumption, investment, and government spending. This data treatment matches the baseline model.

5 Conclusion

The impact of trade integration on relative prices across sectors is a venerable topic in international economics. So too, much as been written about the impacts of globalization on the dynamics of inflation. In this paper, we have brought these two strands of thought together. There is a plausible case that changes in trade have influenced relative inflation across industries. And, in an accounting sense, rising trade restrains consumer price growth relative to growth in the price of domestic value added content (GDP deflators). Nonetheless, we've developed a suite of models to argue that rising trade actually generates inflation.

One important element of our argument is that much of the increase in trade (at least in the US) has been due to rising imports of intermediate inputs. While standard explanations of the impact of trade on inflation emphasize shifts in the Phillips curve, we have demonstrated that rising input trade has no such effect. The second element is that the increase in trade has been spread over time. Anticipated increases in trade – consistent with widespread understanding that globalization was an ongoing process of integration, and that trade agreements were leading to liberalization over time – lead to increases in aggregate demand, which generate inflation. Further, we have also shown that neither changes in capital inflows, nor pro-competitive effects of trade on markups overturn these basic forces, and they may in fact strengthen them. Overall, we are left with the conclusion that trade integration is inflationary.

To conclude, we highlight three topics regarding the nexus between trade and inflation that merit further work. First, while we have conducted our analysis in a small open economy framework, it would be worthwhile to revisit the questions we ask in large open economy models. While we believe our conclusions are robust to this extension, multi-country models that incorporate trade in inputs and final goods would provide fertile ground to study how changes in trade influence inflation synchronization across countries.

Second, our framework adopts a sufficient statistics approach to analyze historical developments in trade. To study prospective shocks – whether to trade policy or other exogenous variables – one needs to solve for the impact of those policies on domestic sourcing shares themselves. This requires taking a stand on the particulars of the shocks (e.g., whether shocks are temporary or permanent, whether they influence trade in inputs or final goods), as well as on features of the model that influence dynamic responses to those shocks (e.g., the currency invoicing of trade). This is a fertile area for future work as well.

Lastly, we have intentionally adopted a very simple approach to characterizing monetary policy, focusing on consumer price inflation targeting. While essentially consistent with how we believe central banks have behaved over this period, one could elaborate on this piece of the model. In unpublished work, we have found our results to be robust to reasonable variants on the policy rule (e.g., including the output gap in the policy rule, incorporating interest rate inertia, etc.). We have not attempted to characterize optimal policy in response to trade shocks, though this would be worthwhile. Farther afield, one might consider making the central bank's desired level of inflation endogenous, as in classic contributions by Romer (1993) and Rogoff (2003), which emphasize that increased trade integration may have reduced the incentives of central banks to inflate the economy. This deserves more consideration outside the US context we have considered, where central bank commitment to low inflation is likely less firm.

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A Import Prices in Theory vs. Data

In Section 1, we suggested that one could (in principle) quantify the role of offshoring in driving costs by looking directly at import price data. We pause here to discuss shortcomings in conventional data sources that preclude this approach, and thus motivate the sufficient-statistic approach that we adopt. While this discussion focuses on US data sources, similar issues arise in virtually all standard national accounts sources.

In Equations 5-7, what we need to quantify the role of inputs in driving costs is the true quality-adjusted cost of imported inputs relative to domestic output. A helpful way to think about this is to re-write the import price as $P_{Ft}(s)=B_{Ft}(s)P_{Ft}^{data}(s)$. Think of the last term $P_{Ft}^{data}(s)$ as the observed import price index (IPI), as measured by the US Bureau of Labor Statistics (BLS) international price program, so then $B_{Ft}(s)$ represents sources of bias that lead measured price indexes to deviate from the (production-cost relevant) measure of import prices required by the model.[34] There are two key sources for this bias.[35]

The first is that existing import price deflators do not capture the full impact of offshoring on the cost of inputs. In a multi-country environment, producers have the ability to substitute among foreign input suppliers (e.g., from Japanese to Chinese suppliers). These cost-saving substitutions are not captured in the multilateral import price indexes produced by the BLS, which implies that measured import prices are likely to biased upward relative to reality, akin to outlet substitution bias in consumer price measurement [Reinsdorf and Yuskavage (2018)]. More broadly, producers also benefit from substituting cheaper foreign for domestic inputs. Because this substitution is not accounted for in measurement of either import prices or domestic purchaser prices by the BLS, it is difficult to estimate the change in the unit cost of the firm's input bundle from existing data. This can be interpreted as a second source of bias in the relative price of imports in our model, which leads the ratio of measured import prices to domestic output prices to understate the decline in costs associated with offshoring.[36]

A second source of bias is that non-price factors may drive substitution between domestic and foreign inputs. A leading concern is that unmeasured improvements in foreign product quality or variety have driven firms to source inputs from abroad. In our application, quality bias is a particularly important concern – the BLS is not able to account for quality improvements in imports using the same methods applied to domestic producer prices. For example, while the BLS applies hedonic adjustments in producer price data, it does not do so for import prices.

A final, somewhat different, challenge in using price data is that firms/industries are linked to one another. Even if import prices were measured perfectly, producers may experience cost reductions either because they directly substitute foreign for domestic ones, or because they buy inputs from upstream suppliers who themselves engage in offshoring. Thus, one also needs input use data to track the propagation of offshoring-induced reductions in costs through the network structure of the economy. This industry-level input use data, collected by the BEA, is not directly comparable with BLS-measured prices in terms of coverage and measurement conventions. Thus, it is challenging to combine import and domestic price data with input-output data, or to aggregate the results to examine the ultimate effect of imported input prices on consumer

[34] Throughout the discussion that follows, we assume that data sources record the quality-adjusted price of domestic output accurately. This assumption is a reasonable way to proceed for two reasons. First, in practice, the BLS performs more extensive quality adjustment in the Producer Price Index (PPI) price program than it does under its Import Price Index program. For example, hedonic price adjustment occurs for producer prices, but not for import prices. Second, rapid changes in the international economy and trading environment point to problems of price measurement as likely being most severe for imports [Houseman and Mandel (2015); Moulton (2018)].

[35] In addition to specific papers cited below, see also Houseman and Mandel (2015) and Moulton (2018).

[36] Put differently, using PPI and IPI data to construct changes in input costs would overstate cost growth of the composite input. Since this bias would lead to the understatement of the quantity of inputs used in production, it also would tend to overstate productivity growth [Houseman et al. (2011)].

or investment goods price levels.

Our approach to circumventing these problems combines data on changes in imported input expenditure shares with a structural model to impute changes in unit costs attributable to offshoring. While enhanced efforts to improve price collection and measurement are needed, the distinct advantage of our approach is that we are able to use “off the self” data from the national accounts to address the macroeconomic impact of offshoring.

B The Three Equation Model

In this appendix, we convert the baseline model in Table 2 into the three equation model. We first derive the Phillips curve, and then we derive the IS Curve. One additional contribution is that we also discuss the relationship between the gross output gap (used in the text) and the output gap for real value added (i.e., actual versus potential GDP).

B.1 Phillips Curve

Domestic price inflation depends on real marginal costs ($\widehat{rmc}_t$): $\pi_{Ht} = \left(\frac{\epsilon-1}{\phi}\right)\widehat{rmc}_t + \beta E_t\left(\pi_{Ht+1}\right)$. We seek to replace real marginal costs with the output gap to obtain a domestic price Phillips curve. We do this first for the output gap defined in terms of gross output, and then discuss replacement with the value-added output gap. Conversion of the domestic price Phillips curve into the consumer price Phillips curve is immediate, recognizing that $\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)}\left(\hat{\lambda}^C_{Ht} - \hat{\lambda}^C_{Ht-1}\right)$. As in the main text, equilibrium objects in the flexible price model have an n in the superscript.

Step One: link $\widehat{rmc}_t$ to the real wage gap. Real marginal costs are: $\widehat{rmc}_t = (1-\alpha)\widehat{rw}_t + \frac{\alpha}{\eta-1}\hat{\lambda}^M_{Ht} - \hat{z}_t$. In the flexible price equilibrium, markups are constant, so real marginal costs are equal to zero: $\widehat{rmc}^n_t = (1-\alpha)\widehat{rw}^n_t + \frac{\alpha}{\eta-1}\hat{\lambda}^M_{Ht} - \hat{z}_t = 0$. Thus, $\widehat{rmc}_t - \widehat{rmc}^n_t = \widehat{rmc}_t$, so we can write:

$$\widehat{rmc}_t = (1-\alpha)\left[\widehat{rw}_t - \widehat{rw}^n_t\right]. \tag{55}$$

Note that the gap between real marginal costs in the actual and flexible price equilibrium only depends on the real wage gap. The direct effects of domestic sourcing of inputs ($\hat{\lambda}^M_{Ht}$) and productivity ($\hat{z}_t$) are differenced away, as they influence real marginal costs the same way in both equilibria.

Step Two: solve for real wage gap. Combining the first order condition for labor supply with labor demand and the definition of real marginal costs, real wages are given by:

$$\widehat{rw}_t = \left(\frac{\alpha\psi}{(1+\alpha\psi)(\eta-1)}\right)\hat{\lambda}^M_{Ht} + \left(\frac{1}{(1+\alpha\psi)(\eta-1)}\right)\hat{\lambda}^C_{Ht} + \left(\frac{\rho}{1+\alpha\psi}\right)\hat{c}_t + \left(\frac{\psi}{1+\alpha\psi}\right)(\hat{y}_t - \hat{z}_t). \tag{56}$$

Evaluating the expression for real wages in the two equilibria, the real wage gap is then:

$$\widehat{rw}_t - \widehat{rw}^n_t = \left(\frac{\rho}{1+\alpha\psi}\right)\left[\hat{c}_t - \hat{c}^n_t\right] + \left(\frac{\psi}{1+\alpha\psi}\right)\left[\hat{y}_t - \hat{y}^n_t\right] \tag{57}$$

$$= \left(\frac{1}{1+\alpha\psi}\right)\left[\hat{q}_t - \hat{q}^n_t\right] + \left(\frac{\psi}{1+\alpha\psi}\right)\left[\hat{y}_t - \hat{y}^n_t\right], \tag{58}$$

where the second line uses the risk sharing condition to eliminate the consumption gap.[37] This says the gap in real wages depends on differences in the real exchange rate and output across equilibria. Note again that that the direct effect of changes in domestic sourcing and productivity drop away, since they move real wages in identical ways in the two equilibria.

[37]Note that changes in foreign consumption $\hat{c}^*_t$ do not appear, because they are differenced away when looking at the consumption gap.

Step Three: solve for the real exchange rate gap. The next step is to swap out the real exchange rate gap, using the goods market clearing condition to link the real exchange rate gap to the output gap and real marginal costs. Collecting pieces, the goods market clearing condition is:

$$\hat{y}_t = s_C \left[\frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \hat{c}^*_t + \frac{1}{\rho}\hat{q}_t\right] + s_M \left[\frac{\eta}{\eta-1}\hat{\lambda}^M_{Ht} + \widehat{rmc}_t + \hat{y}_t - \frac{1}{\eta-1}\hat{\lambda}^M_{Ht}\right] + s_X \left[\frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \eta\hat{q}_t + \hat{c}^*_t\right]. \tag{59}$$

We rearrange to isolate output on the left-hand side:

$$\hat{y}_t = \frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \hat{c}^*_t + \left(\frac{s_C/\rho + s_X\eta}{1-s_M}\right)\hat{q}_t + \left(\frac{s_M}{1-s_M}\right)\hat{\lambda}^M_{Ht} + \left(\frac{s_M}{1-s_M}\right)\widehat{rmc}_t, \tag{60}$$

where $s_C \equiv \frac{C_0}{Y_0}$, $s_M \equiv \frac{M_0}{Y_0}$, $s_X \equiv \frac{X_0}{Y_0}$, so $s_C + s_M + s_X = 1$.

Evaluating output at the flexible price equilibrium, and taking differences, the gross output gap is:

$$\hat{y}_t - \hat{y}^n_t = \left(\frac{s_C/\rho + s_X\eta}{1-s_M}\right)[\hat{q}_t - \hat{q}^n_t] + \left(\frac{s_M}{1-s_M}\right)\widehat{rmc}_t. \tag{61}$$

Solving for the real exchange rate gap gives us:

$$\hat{q}_t - \hat{q}^n_t = \left(\frac{1-s_M}{s_C/\rho + s_X\eta}\right)[\hat{y}_t - \hat{y}^n_t] - \left(\frac{s_M}{s_C/\rho + s_X\eta}\right)\widehat{rmc}_t. \tag{62}$$

Step Four: Link real wages and output gap. Plugging the expression for the real exchange rate gap into the real wage gap equation yields:

$$\begin{aligned}\widehat{rw}_t - \widehat{rw}^n_t &= \left(\frac{1}{1+\alpha\psi}\right)\left(\frac{1-s_M}{s_C/\rho + s_X\eta}\right)[\hat{y}_t - \hat{y}^n_t] - \left(\frac{1}{1+\alpha\psi}\right)\left(\frac{s_M}{s_C/\rho + s_X\eta}\right)\widehat{rmc}_t + \left(\frac{\psi}{1+\alpha\psi}\right)[\hat{y}_t - \hat{y}^n_t] \\ &= \left[\left(\frac{1}{1+\alpha\psi}\right)\left(\frac{1-s_M}{s_C/\rho + s_X\eta}\right) + \left(\frac{\psi}{1+\alpha\psi}\right)\right][\hat{y}_t - \hat{y}^n_t] - \left(\frac{1-\alpha}{1+\alpha\psi}\right)\left(\frac{s_M}{s_C/\rho + s_X\eta}\right)[\widehat{rw}_t - \widehat{rw}^n_t],\end{aligned} \tag{63}$$

where the second line eliminates real marginal costs using Equation 55. Then, we solve for the real wage gap:

$$\widehat{rw}_t - \widehat{rw}^n_t = \chi[\hat{y}_t - \hat{y}^n_t], \tag{64}$$

where $\chi \equiv \left[1 + \left(\frac{1-\alpha}{1+\alpha\psi}\right)\left(\frac{s_M}{s_C/\rho+s_X\eta}\right)\right]^{-1}\left[\left(\frac{1}{1+\alpha\psi}\right)\left(\frac{1-s_M}{s_C/\rho+s_X\eta}\right) + \left(\frac{\psi}{1+\alpha\psi}\right)\right]$. It is straightforward to verify that $\chi > 0$ under the parameter restrictions imposed in the main text.

Step Five: Link real marginal costs and output gap. Plugging back into Equation 55, real marginal costs are linked to the gross output gap:

$$\widehat{rmc}_t = (1-\alpha)\chi[\hat{y}_t - \hat{y}^n_t]. \tag{65}$$

A positive output gap yields an increase in real marginal costs, since $\alpha \in (0,1)$ and $\chi > 0$.

Step Six: Write down the domestic price Phillips Curve. Substituting for real marginal costs in the domestic price inflation equation gives us the domestic price Phillips curve:

$$\pi_{Ht} = \left(\frac{(\epsilon-1)(1-\alpha)\chi}{\phi}\right)[\hat{y}_t - \hat{y}_t^n] + \beta E_t(\pi_{Ht+1}). \tag{66}$$

Since $\epsilon > 1$ and $\phi > 0$, a positive gross output gap pushes up domestic price inflation, conditional on expected future inflation.

Step Seven (Optional): replace gross output gap with value-added output gap. Equation 65 links real marginal costs to the gross output gap, which results in a domestic price Phillips curve that depends on the gross output gap. Often, the output gap is defined in terms of real value added (GDP), rather than gross output. So we pause here to demonstrate how to write the Phillips curve in terms of real value added.

In our model, real GDP can be constructed via double deflation, as in the national accounts:

$$\widehat{rva}_t = \left(\frac{P_{H0}Y_0}{GDP_0}\right)\hat{y}_t - \left(\frac{P_{M0}M_0}{GDP_0}\right)\hat{m}_t, \tag{67}$$

where $\widehat{rva}_t$ is the log deviation in real value added from steady state and $GDP_0 = P_{H0}Y_0 - P_{M0}M_0$ is value added in the steady state. Using this result, the value-added output gap is:

$$\widehat{rva}_t - \widehat{rva}_t^n = \left(\frac{1}{s_{VA}}\right)[\hat{y}_t - \hat{y}_t^n] - \left(\frac{1}{s_{VA}} - 1\right)[\hat{m}_t - \hat{m}_t^n], \tag{68}$$

where $s_{VA} \equiv \frac{GDP_0}{P_{H0}Y_0}$. Input use in the flexible price equilibrium is given by $\hat{m}_t^n = \hat{y}_t^n - \frac{1}{\eta-1}\hat{\lambda}_{Ht}^M$, thus:

$$\hat{m}_t - \hat{m}_t^n = \widehat{rmc}_t + [\hat{y}_t - \hat{y}_t^n]. \tag{69}$$

Plugging back gives us:

$$\begin{aligned}
\widehat{rva}_t - \widehat{rva}_t^n &= \left(\frac{1}{s_{VA}}\right)[\hat{y}_t - \hat{y}_t^n] - \left(\frac{1}{s_{VA}} - 1\right)[\widehat{rmc}_t + [\hat{y}_t - \hat{y}_t^n]] \\
&= [\hat{y}_t - \hat{y}_t^n] - \left(\frac{1-s_{VA}}{s_{VA}}\right)\widehat{rmc}_t.
\end{aligned} \tag{70}$$

Rewriting yields $\hat{y}_t - \hat{y}_t^n = [\widehat{rva}_t - \widehat{rva}_t^n] + \left(\frac{1-s_{VA}}{s_{VA}}\right)\widehat{rmc}_t$, and we insert this into real marginal costs to get:

$$\widehat{rmc}_t = \left[1 - (1-\alpha)\chi\left(\frac{1-s_{VA}}{s_{VA}}\right)\right]^{-1}(1-\alpha)\chi[\widehat{rva}_t - \widehat{rva}_t^n] \tag{71}$$

The domestic price Phillips Curve is then given by:

$$\pi_{Ht} = \left(\frac{(\epsilon-1)(1-\alpha)\chi}{\phi}\right)\left[1 - (1-\alpha)\chi\left(\frac{1-s_{VA}}{s_{VA}}\right)\right]^{-1}[\widehat{rva}_t - \widehat{rva}_t^n] + \beta E_t(\pi_{Ht+1}). \tag{72}$$

B.2 IS Curve

As usual, derivation of the IS curve starts with the Euler Equation. We first convert the Euler Equation into an IS curve that relates the gross output gap to a real interest rate gap. For completeness, we also rewrite the IS curve in terms of real value added. To fully characterize the IS curve, we then solve for the natural real interest rate.

Step One: write Euler Equation with consumption and interest rate gaps. Start with the Euler equation: $\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\rho}\left(\hat{r}_t - E_t\pi_{Ct+1}\right)$, and take differences between the actual and flexible price equilibria:

$$\hat{c}_t - \hat{c}_t^n = E_t\left[\hat{c}_{t+1} - \hat{c}_{t+1}^n\right] - \frac{1}{\rho}\left[\hat{\tilde{r}}_t - \hat{\tilde{r}}_t^n\right], \tag{73}$$

where $\hat{\tilde{r}}_t \equiv \hat{r}_t - E_t\pi_{Ct+1}$ and $\hat{\tilde{r}}_t^n \equiv \hat{r}_t^n - E_t\pi_{Ct+1}^n = \hat{r}_t^n - \frac{1}{(\eta-1)}E_t\Delta\ln\lambda_{Ht+1}^C$.

Step Two: Link consumption and output gaps. We use the goods market clearing condition to link the consumption and output gaps. The market clearing condition can be written as:

$$[1 - s_M]\,\hat{y}_t = s_C\left[\frac{\eta}{\eta-1}\hat{\lambda}_{Ht}^C + \hat{c}_t\right] + s_M\left[\hat{\lambda}_{Ht}^M + \widehat{rmc}_t\right] + s_X\left[\frac{\eta}{\eta-1}\hat{\lambda}_{Ht}^C + \eta\hat{q}_t + \hat{c}_t^*\right]. \tag{74}$$

Then we take differences between actual and flexible price equilibria:

$$[1 - s_M]\left[\hat{y}_t - \hat{y}_t^n\right] = s_C\left[\hat{c}_t - \hat{c}_t^n\right] + s_M\widehat{rmc}_t + s_X\eta\left[\hat{q}_t - \hat{q}_t^n\right], \tag{75}$$

where again the direct effects of changes in domestic sourcing and foreign consumption drop away. We proceed to eliminate real marginal costs using Equation 65 and rearrange:

$$\hat{c}_t - \hat{c}_t^n = \left[\frac{(1-s_M) - s_M(1-\alpha)\chi}{s_C}\right]\left[\hat{y}_t - \hat{y}_t^n\right] - \frac{s_X\eta}{s_C}\left[\hat{q}_t - \hat{q}_t^n\right]. \tag{76}$$

And then we can combine Equation 62 and 65 to eliminate the real exchange rate gap:

$$\begin{aligned}\hat{c}_t - \hat{c}_t^n &= \left[\frac{(1-s_M) - s_M(1-\alpha)\chi}{s_C}\right]\left[\hat{y}_t - \hat{y}_t^n\right] - \frac{s_X\eta}{s_C}\left(\frac{1-s_M}{s_C/\rho + s_X\eta} - \left(\frac{s_M(1-\alpha)\chi}{s_C/\rho + s_X\eta}\right)\right)\left[\hat{y}_t - \hat{y}_t^n\right] \\ &= \theta\left[\hat{y}_t - \hat{y}_t^n\right],\end{aligned} \tag{77}$$

where $\theta \equiv \frac{(1-s_M)-s_M(1-\alpha)\chi}{s_C} - \frac{s_X\eta}{s_C}\left(\frac{1-s_M}{s_C/\rho+s_X\eta} - \frac{s_M(1-\alpha)\chi}{s_C/\rho+s_X\eta}\right) > 0$.

Step Three: Define IS Curve We replace the consumption gap with the output gap in the Euler Equation and rearrange to get the IS curve for gross output:

$$\hat{y}_t - \hat{y}_t^n = E_t\left[\hat{y}_{t+1} - \hat{y}_{t+1}^n\right] - \frac{1}{\theta\rho}\left(\hat{\tilde{r}}_t - \hat{\tilde{r}}_t^n\right). \tag{78}$$

Step Four (Optional): Replace gross output gap with value-added output gap. Reusing results in Equations 70-71 above, we can link the gross output gap and the value-added output gap:

$$\hat{y}_t - \hat{y}_t^n = \Phi\left[\widehat{rva}_t - \widehat{rva}_t^n\right], \tag{79}$$

where $\Phi \equiv \left[1+\left[\frac{s_{VA}}{1-s_{VA}}-(1-\alpha)\chi\right]^{-1}(1-\alpha)\chi\right]>0$. Then substituting into the IS curve, we get:

$$\widehat{rva}_t - \widehat{rva}_t^n = E_t\left[\widehat{rva}_{t+1} - \widehat{rva}_{t+1}^n\right] - \frac{1}{\Phi\theta\rho}\left(\hat{\hat{r}}_t - \hat{\hat{r}}_t^n\right). \tag{80}$$

Thus, as in the Phillips curve, the translation from gross output to real value added gaps only influences the slope of the IS curve.

B.3 The Natural Real Interest Rate

The natural real interest rate is pinned down in the flexible price equilibrium by the Euler Equation:

$$\hat{\hat{r}}_t^n = \rho E_t\left[\hat{c}_{t+1}^n - \hat{c}_t^n\right]. \tag{81}$$

We proceed here to solve for the natural real rate by pinning down consumption growth in the flexible price equilibrium.

Step One: Link consumption to real exchange rate. Under the complete markets assumption, $\hat{c}_t^n = \hat{c}_t^* + \frac{1}{\rho}\hat{q}_t^n$, so we can write:

$$\hat{\hat{r}}_t^n = \rho E_t\left[\hat{c}_{t+1}^* - \hat{c}_t^*\right] + E_t\left(\hat{q}_{t+1}^n - \hat{q}_t^n\right) \tag{82}$$

Step Two: Link real exchange rate to output dynamics. The market clearing condition in the flexible price equilibrium is:

$$\hat{y}_t^n = \frac{\eta}{\eta-1}\hat{\lambda}_{Ht}^C + \hat{c}_t^* + \left(\frac{s_C/\rho + s_X\eta}{1-s_M}\right)\hat{q}_t^n + \left(\frac{s_M}{1-s_M}\right)\hat{\lambda}_{Ht}^M. \tag{83}$$

Take differences of this equation and rearrange to get:

$$\Delta\hat{q}_t^n = \left(\frac{1-s_M}{s_C/\rho + s_X\eta}\right)\left[\Delta\hat{y}_t^n - \frac{\eta}{\eta-1}\Delta\hat{\lambda}_{Ht}^C - \left(\frac{s_M}{1-s_M}\right)\Delta\hat{\lambda}_{Ht}^M - \Delta\hat{c}_t^*\right] \tag{84}$$

Note that the phase in of the shocks directly raises $\Delta\hat{q}_t^n$, and if they lead to a boom in output growth (yielding $\Delta\hat{y}_{t+1}^n > 0$) then push it up further.

Step Three: Pin down output growth. From the supply side of the flexible price equilibrium, we know that $\hat{y}_t^n = \hat{z}_t + (1-\alpha)\hat{l}_t^n + \alpha\hat{m}_t^n$ and $\hat{m}_t^n = \hat{y}_t^n - \frac{1}{\eta-1}\hat{\lambda}_{Ht}^M$, so output can be expressed as:

$$\hat{y}_t^n = \frac{1}{1-\alpha}\hat{z}_t + \hat{l}_t^n - \frac{\alpha}{(\eta-1)(1-\alpha)}\hat{\lambda}_{Ht}^M. \tag{85}$$

Then we can pin down $\hat{l}_t^n$ using labor supply and the real wage (obtained from $\widehat{rmc}_t^n = 0$):

$$\hat{l}_t^n = -\frac{\rho}{\psi}\hat{c}_t^* - \frac{1}{\psi}\hat{q}_t^n - \frac{\alpha}{\psi(1-\alpha)(\eta-1)}\hat{\lambda}_{Ht}^M - \frac{1}{\psi(\eta-1)}\hat{\lambda}_{Ht}^C + \left(\frac{1}{\psi(1-\alpha)}\right)\hat{z}_t \tag{86}$$

Combining these two equations and taking differences gives us:

$$\Delta\hat{y}_t^n = \left(1+\frac{1}{\psi}\right)\left(\frac{1}{1-\alpha}\right)\Delta\hat{z}_t - \frac{\alpha}{(\eta-1)(1-\alpha)}\left[1+\frac{1}{\psi}\right]\Delta\hat{\lambda}_{Ht}^M - \frac{\rho}{\psi}\Delta\hat{c}_t^* - \frac{1}{\psi}\Delta\hat{q}_t^n - \frac{1}{\psi(\eta-1)}\Delta\hat{\lambda}_{Ht}^C. \quad (87)$$

Step Four: Combine steps to solve for natural real interest rate. We can combine results from steps two, three, and four to write the natural real interest rate as a function of exogenous shocks:

$$\hat{r}_t^n = \varOmega_{C^*}E_t\Delta\hat{c}_{t+1}^* + \varOmega_Z E_t\Delta\hat{z}_{t+1} + \varOmega_M E_t\Delta\hat{\lambda}_{Ht+1}^M + \varOmega_C E_t\Delta\hat{\lambda}_{Ht+1}^C, \quad (88)$$

where $\varOmega_{C^*} = \frac{\psi(\rho-1)}{1+\psi} > 0$, $\varOmega_Z = \frac{1}{1-\alpha} > 0$, $\varOmega_M = -\left[\frac{\alpha}{(\eta-1)(1-\alpha)} + \frac{\psi s_M}{(1+\psi)(1-s_M)}\right] < 0$, and $\varOmega_C = -\frac{1+\psi\eta}{(1+\psi)(\eta-1)} < 0$.

C Model with Physical Capital

In this section, we provide an extension of the baseline model that introduces physical capital in production. We first describe the model, and then compute simulated inflation in the model. This serves to demonstrate the robustness of the key findings to the inclusion of physical capital in the model.

C.1 Model Equations

We make four modifications to the baseline model. First, we modify the production function to include capital as a primary factor. Second, we introduce a law of motion for the capital and specify the production function and market structure for investment. Specifically, investment is produced competitively from domestic output, subject to costs of adjusting the capital stock as in Hayashi (1982). Third, we derive an optimal investment condition by which marginal costs of creating a unit of capital is equal to the shadow price of capital. Fourth, we impose an arbitrage condition that equates the expected rate of return to capital to the real interest rate in the economy.

Formally, the aggregate production function is now:

$$Y_t = Z_t(L_t^{\varrho} K_t^{1-\varrho})^{1-\alpha} M_t^{\alpha}. \tag{89}$$

The law of motion for capital is:

$$K_t = K_{t-1}(1-\delta) + I_t\left[1 - \varphi(I_t/K_{t-1})\right], \tag{90}$$

where the function $\varphi(\cdot)$ captures the cost of adjustment the capital stock, and it satisfies the following properties: $\varphi(\delta) = 0$, $\varphi'(\delta) = 0$, and $\varphi''(\cdot) > 0$. We assume that investment is produced using home goods, so the market clearing condition for home goods now becomes:

$$Y_{Ht} = C_{Ht} + M_{Ht} + I_t(1 + \varphi(I_t/K_{t-1})) + X_t + \frac{\phi}{2}\left(\frac{P_{Ht}}{P_{H,t-1}} - 1\right)^2 Y_t \tag{91}$$

Since the price of one unit of investment is P_{Ht}, optimal investment implies that the shadow price of a capital good, P_{Kt}, is equal to the marginal cost of creating it:

$$P_{Kt}(1 - \varphi(I_t/K_{t-1}) - \varphi'(I_t/K_{t-1})I_t/K_{t-1}) = P_{Ht} \tag{92}$$

Finally, arbitrage implies that the rate of return of capital satisfies:

$$E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_{Ct}}{P_{C,t+1}}(1+i_t)\right] = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_{Ct}P_{H,t+1}}{P_{C,t+1}P_{H,t}}\frac{(1-\varrho)(1-\alpha)\frac{MC_{t+1}Y_{t+1}}{P_{Ht+1}K_{t+1}} + \frac{P_{Kt+1}}{P_{Ht+1}}(1-\delta)}{\frac{P_{Kt}}{P_{Ht}}}\right] \tag{93}$$

The log-linearized model with capital is presented in Table 4. The notation generally follows the baseline model. We add four new variables related to the cost of capital and capital acumulation, where hats denote log deviations from steady state: $\hat{i}_t$ is physical capital investment, $\hat{k}_t$ is the capital stock , $\widehat{rpk}_t$ is the relative price of capital (relative to home output), and $\widehat{rd}_t$ is the real rental rate of capital. The equilibrium system has 17 endogenous variables, including prices $\{\widehat{rw}_t, \widehat{rd}_t, \hat{r}_t, \widehat{rpk}_t, \widehat{rmc}_t, \hat{q}_t, \hat{\pi}_{Ht}, \hat{\pi}_{Ct}\}$ and quantities $\{\hat{y}_t, \hat{m}_t, \hat{l}_t, \hat{m}_{Ht}, \hat{k}_t, \hat{i}_t, \hat{c}_{Ht}, \hat{c}_t, \hat{x}_t\}$ and 4 exogenous variables $\{\hat{\lambda}^C_{Ht}, \hat{\lambda}^M_{Ht}, \hat{c}^*_t, \hat{z}_t\}$.

Table 4: Log-Linearization of the Model with Capital

Consumption-Leisure	$\hat{l}_t = -\frac{\rho}{\psi}\hat{c}_t + \frac{1}{\psi}\widehat{rw}_t - \frac{1}{\psi(\eta-1)}\hat{\lambda}^C_{Ht}$
Consumption Allocation	$\hat{c}_{Ht} = \frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \hat{c}_t$
Euler Equation	$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\rho}\left(\hat{r}_t - E_t\pi_{Ct+1}\right)$
Input Choices	$\hat{l}_t = \widehat{rmc}_t + \hat{y}_t - \widehat{rw}_t$ $\hat{m}_t = \widehat{rmc}_t + \hat{y}_t - \frac{1}{\eta-1}\hat{\lambda}^M_{Ht}$ $\hat{m}_{Ht} = \frac{\eta}{\eta-1}\hat{\lambda}^M_{Ht} + \hat{m}_t$ $\hat{k}_t = -(\bar{i}\widehat{rd}_t + \delta\widehat{rpk}_t)/(\bar{i}+\delta) + \widehat{rmc}_t + \hat{y}_t$
Production Function	$\hat{y}_t = \hat{z}_t + (1-\alpha)\left((1-\varrho)\hat{k}_t + \varrho\hat{l}_t\right) + \alpha\hat{m}_t$
Law of Motion for Capital	$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_{t-1}$
Optimal Investment	$\widehat{rpk}_t = \overline{ac} * \left(\hat{i}_t - \hat{k}_{t-1}\right)$
Domestic Price Inflation	$\pi_{Ht} = \left(\frac{\epsilon-1}{\phi}\right)\widehat{rmc}_t + \beta E_t\left(\pi_{Ht+1}\right)$
Consumer Price Index	$\pi_{Ct} = \pi_{Ht} + \frac{1}{(\eta-1)}\left(\hat{\lambda}^C_{Ht} - \hat{\lambda}^C_{Ht-1}\right)$
Market Clearing	$\hat{y}_t = \left(\frac{C_{H0}}{Y_0}\right)\hat{c}_{Ht} + \left(\frac{M_{H0}}{Y_0}\right)\hat{m}_{Ht} + \frac{I}{Y}\hat{i}_t + \left(\frac{X_0}{Y_0}\right)\hat{x}_t$ $\hat{x}_t = \frac{\eta}{\eta-1}\hat{\lambda}^C_{Ht} + \eta\hat{q}_t + \hat{c}^*_t$ $\hat{c}_t = \hat{c}^*_t + \frac{1}{\rho}\hat{q}_t$
Arbitrage	$r_t + \widehat{rpk}_t - E_t\left[\pi_{Ht+1}\right] = E_t\left[\frac{\bar{i}+\delta}{1+i}\left(\widehat{rmc}_{t+1} + \hat{y}_{t+1} - \hat{k}_{t+1}\right) + \frac{1-\delta}{1+i}\widehat{rpk}_{t+1}\right]$
Monetary Policy Rule	$\hat{r}_t = \omega\pi_{Ct}$

C.2 Inflation Dynamics with Capital Accumulation

To simulate the model with capital, we need to calibrate two new parameters. We set the capital share in value added, $(1-\varrho)$ to 0.4. Note that our baseline model is the limit of the model with capital when $(1-\varrho)$ tends to 0. We set the adjustment cost parameter, $\overline{ac}$,[38] to 0.5, which implies a standard deviation for the price of capital that is half the standard deviation of investment. This is close to the relative standard deviation in U.S. data; At business cycle frequencies, it is around 1/3, but it increases to around 1/2 when medium-term frequencies are included in the analysis [Comin and Gertler (2006)].

Figure 11a presents the simulated inflation series after feeding in the observed home sourcing shares in the model with capital. For comparison purposes we also plot the simulated inflation series in the baseline model. Figure 11b separates the inflation series resulting from changes in home sourcing shares for final consumption goods and for intermediate goods.

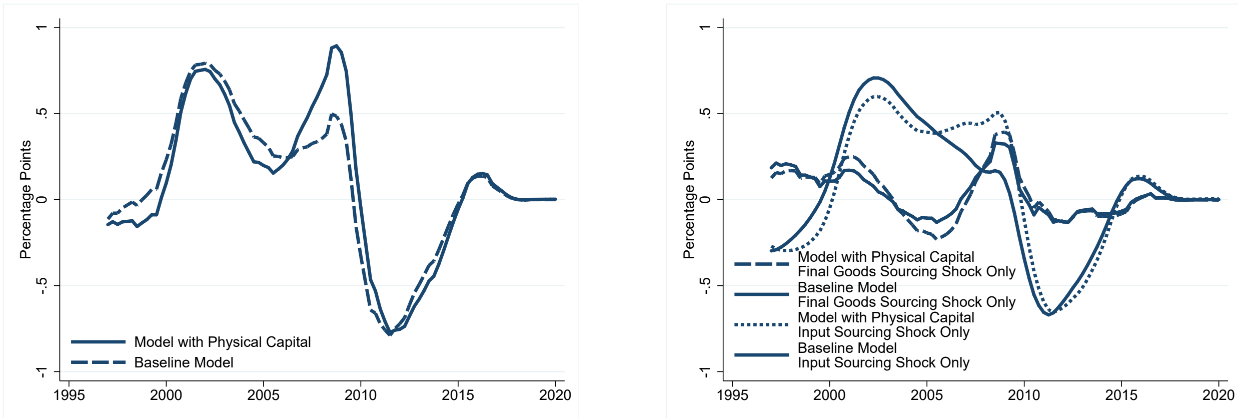
The main takeaway is that the dynamics of inflation in the model with capital are very similar to the baseline. Inflation increases in response to the decline in home sourcing during the first half of the simulation period, and then declines in the latter half as home sourcing of inputs reverts, much like the

[38] $\overline{ac}$ is equal to $\varphi''(I_0/K_0) * (I_0/K_0)^2$,where I_0/K_0 denotes the investment to output ratio in steady state.

Figure 11: Simulation of Inflation in the Model with Physical Capital

(a) Simulated Inflation

(b) Simulated Inflation for Individual Shocks



baseline model. The main difference vis-à-vis the baseline concerns dynamics around the Great Recession, where input sourcing shocks push inflation up more in the model with capital than the baseline model. This is a somewhat tangential feature of the data, on which we place relatively little weight.

D Three Equation Model with Capital Inflow Shocks

As described in the text, the model with capital inflow shocks makes one change to the baseline model (summarized in Tables 1-2). We drop the complete markets assumption, and we thus replace the risk sharing condition with with a trade deficit equation given by:

$$TDY_t\left(P_{Ht}Y_t\right) = P_{Ft}\tau_{Ct}C_{Ft} + P_{Ft}\tau_{Mt}M_{Ft} - P_{Ht}X_t, \tag{94}$$

where $TDY_t = \frac{TD_t}{P_{Ht}Y_t}$ is the trade deficit as a share of gross output, which is treated as an additional exogenous variable (i.e., shock). This expression can be log-linearized as follows:

$$TD_0\left(\widehat{tdy}_t + \hat{y}_t\right) = IM_0^C\left(\left(\frac{1-\eta\lambda_{H0}^C}{(\eta-1)(1-\lambda_{H0}^C)}\right)\hat{\lambda}_{Ht}^C + \hat{c}_t\right) + IM_0^M\left(\left(\frac{1-\lambda_{H0}^M}{(\eta-1)(1-\lambda_{H0}^M)}\right)\hat{\lambda}_{Ht}^M + \hat{m}_t\right) - EX_0\hat{x}_t, \tag{95}$$

where we used $\hat{\lambda}_{Ft}^C = -\left(\frac{\lambda_{H0}^C}{1-\lambda_{H0}^C}\right)\hat{\lambda}_{Ht}^C$, $\hat{\lambda}_{Ft}^M = -\left(\frac{\lambda_{H0}^M}{1-\lambda_{H0}^M}\right)\hat{\lambda}_{Ht}^M$, $\hat{\lambda}_{Ht}^C = (1-\eta)\left(\hat{p}_{Ht} - \hat{p}_{Ct}\right)$, and $\hat{\lambda}_{Ht}^M = (1-\eta)\left(\hat{p}_{Ht} - \hat{p}_{Mt}\right)$ to rewrite the expression. Further, $TD_0 \equiv TDY_0\left(P_{H0}Y_0\right)$ is the trade deficit, $IM_0^C \equiv P_{F0}\tau_{C0}C_{F0}$ is the value of imports for final consumption , $IM_0^M \equiv P_{F0}\tau_{M0}M_{F0}$ is the value of imported inputs, and $EX_0 \equiv P_{H0}X_0$ is the value of exports, all evaluated at the date zero steady state.

We now proceed to use this trade deficit condition in place of the risk sharing condition to re-derive the Phillips and IS curves, as well as solve for the real natural rate of interest. These derivations support the discussion in main text.

D.1 Phillips Curve

Referring back to Appendix B, Step One and Step Two of the derivation are identical. Starting with Step Three, we need to solve for the real exchange rate gap. The goods market clearing condition is:

$$\hat{y}_t = s_C\left[\frac{\eta}{\eta-1}\hat{\lambda}_{Ht}^C + \hat{c}_t\right] + s_M\left[\frac{\eta}{\eta-1}\hat{\lambda}_{Ht}^M + \widehat{rmc}_t + \hat{y}_t - \frac{1}{\eta-1}\hat{\lambda}_{Ht}^M\right] + s_X\left[\frac{\eta}{\eta-1}\hat{\lambda}_{Ht}^C + \eta\hat{q}_t + \hat{c}_t^*\right]. \tag{96}$$

Evaluating this expression in the flexible price equilibrium and taking differences gives us:

$$\hat{y}_t - \hat{y}_t^n = \left(\frac{s_C}{1-s_M}\right)\left(\hat{c}_t - \hat{c}_t^n\right) + \left(\frac{s_M}{1-s_M}\right)\widehat{rmc}_t + \left(\frac{s_X\eta}{1-s_M}\right)\left(\hat{q}_t - \hat{q}_t^n\right). \tag{97}$$

In Appendix B, we eliminated consumption using the risk sharing condition. Here we will use the trade deficit equation instead. Using Equation 95, we can substitute out for $\hat{m}_t$ and $\hat{x}_t$, and then rearrange to get:

$$\begin{aligned} TD_0\widehat{tdy}_t + \left(TD_0 - IM_0^M\right)\hat{y}_t = &\left[IM_0^C\left(\frac{1-\eta\lambda_{H0}^C}{(\eta-1)(1-\lambda_{H0}^C)}\right) - EX_0\left(\frac{\eta}{\eta-1}\right)\right]\hat{\lambda}_{Ht}^C + IM_0^C\hat{c}_t \\ &+ IM_0^M\left(\frac{1-\lambda_{H0}^M}{(\eta-1)(1-\lambda_{H0}^M)} - \frac{1}{\eta-1}\right)\hat{\lambda}_{Ht}^M + IM_0^M\widehat{rmc}_t - EX_0\eta\hat{q}_t - EX_0\hat{c}_t^*. \end{aligned} \tag{98}$$

Evaluate this expression at the flexible price equilibrium, where $\widehat{rmc}_t^n = 0$, and take differences to obtain:

$$\left(TD_0 - IM_0^M\right)\left(\hat{y}_t - \hat{y}_t^n\right) = IM_0^C\left(\hat{c}_t - \hat{c}_t^n\right) + IM_0^M\widehat{rmc}_t - EX_0\eta\left(\hat{q}_t - \hat{q}_t^n\right). \tag{99}$$

Note there is no direct effect of the trade deficit here, because it is assumed to be the same in the actual and flexible price equilibria. The steady state trade deficit matters for determination of the coefficients in this equation, but the main difference from the baseline model is that the real exchange rate gap is no longer linked to consumption via risk sharing.

Equations 97 and 99 allow us to solve for the real exchange rate gap in terms of the gross output gap and real marginal costs:

$$(\hat{q}_t - \hat{q}_t^n) = \left(\frac{IM_0^M + IM_0^C \left(\frac{1-s_M}{s_C}\right) - TD_0}{IM_0^C \left(\frac{s_X \eta}{s_C}\right) + EX_0 \eta} \right) (\hat{y}_t - \hat{y}_t^n) + \left(\frac{IM_0^M - IM_0^C \left(\frac{s_M}{s_C}\right)}{IM_0^C \left(\frac{s_X \eta}{s_C}\right) + EX_0 \eta} \right) \widehat{rmc}_t \tag{100}$$

Proceeding to Step Four, we need to link real wages and the output gap. Starting from Equation 57, we use the Equation 97 to replace the consumption gap:

$$\begin{aligned}
\widehat{rw}_t - \widehat{rw}_t^n &= \left(\frac{\rho}{1+\alpha\psi} \right) [\hat{c}_t - \hat{c}_t^n] + \left(\frac{\psi}{1+\alpha\psi} \right) [\hat{y}_t - \hat{y}_t^n] \\
&= \left(\frac{\rho(1-s_M)}{(1+\alpha\psi)s_C} + \frac{\psi}{1+\alpha\psi} \right) (\hat{y}_t - \hat{y}_t^n) - \left(\frac{\rho}{1+\alpha\psi} \frac{s_M}{s_C} \right) \widehat{rmc}_t - \frac{\rho\eta}{1+\alpha\psi} \frac{s_X}{s_C} (\hat{q}_t - \hat{q}_t^n), \qquad (101)
\end{aligned}$$

where $\hat{c}_t - \hat{c}_t^n = \frac{1-s_M}{s_C} (\hat{y}_t - \hat{y}_t^n) - \frac{s_M}{s_C} \widehat{rmc}_t - \eta \frac{s_X}{s_C} (\hat{q}_t - \hat{q}_t^n)$. We then use Equation 55 to replace $\widehat{rmc}_t$, and rearrange to obtain:

$$(\widehat{rw}_t - \widehat{rw}_t^n) = \left(\frac{\rho(1-s_M) + \psi s_C}{(1+\alpha\psi)\, s_C + \rho(1-\alpha) s_M} \right) (\hat{y}_t - \hat{y}_t^n) - \frac{\rho\eta s_X}{(1+\alpha\psi)\, s_C + \rho(1-\alpha) s_M} (\hat{q}_t - \hat{q}_t^n) \tag{102}$$

Then we combine Equations 100, 102, and 55 to write the real wage gap as a function of the output gap:

$$\widehat{rw}_t - \widehat{rw}_t^n = \bar{\chi}\dot{\chi}\, (\hat{y}_t - \hat{y}_t^n) \tag{103}$$

$$\text{with} \quad \bar{\chi} \equiv \left[1 + \frac{\rho\eta s_X (1-\alpha)}{(1+\alpha\psi)\, s_C + \rho(1-\alpha) s_M} \left(\frac{IM_0^M - IM_0^C \left(\frac{s_M}{s_C}\right)}{IM_0^C \left(\frac{s_X \eta}{s_C}\right) + EX_0 \eta} \right) \right]^{-1}$$

$$\text{and} \quad \dot{\chi} \equiv \frac{1}{(1+\alpha\psi)\, s_C + \rho(1-\alpha) s_M} \left[\rho(1-s_M) + \psi s_C - \rho\eta s_X \left(\frac{IM_0^M + IM_0^C \left(\frac{1-s_M}{s_C}\right) - TD_0}{IM_0^C \left(\frac{s_X \eta}{s_C}\right) + EX_0 \eta} \right) \right].$$

In Step Five, we write real marginal costs as a function of the output gap:

$$\widehat{rmc}_t = (1-\alpha)\bar{\chi}\dot{\chi}\, [\hat{y}_t - \hat{y}_t^n]. \tag{104}$$

And finally, this gives us the domestic price Phillips curve:

$$\pi_{Ht} = \left(\frac{(\epsilon-1)(1-\alpha)\bar{\chi}\dot{\chi}}{\phi} \right) [\hat{y}_t - \hat{y}_t^n] + \beta E_t (\pi_{Ht+1}) \tag{105}$$

The end result of this analysis is that the domestic price Phillips curve is nearly identical to the baseline model with complete markets, but for a change in the slope of the curve. The consumer price Phillips curve then inherits this modest difference. Importantly, neither the domestic sourcing shocks, nor the capital inflow shocks shift the domestic price Phillips curve. As a result, only the domestic sourcing shocks for final goods will appear in the consumer price Phillips Curve, as in Equation 34.

D.2 IS Curve

Referring back to Appendix B, Step One of the derivation of the IS curve is identical. In Step Two, we link consumption and output gaps. Starting from Equation 97, we plug in for the real exchange rate gap using Equation 100, and then evaluate real marginal costs using Equation 104. Rearranging the result yields:

$$\hat{c}_t - \hat{c}_t^n = \bar{\theta}\left(\hat{y}_t - \hat{y}_t^n\right), \tag{106}$$

with $\bar{\theta} \equiv \left(\frac{1-s_M}{s_C}\right)\left[1 - \left[\left(\frac{s_M}{1-s_M}\right) + \left(\frac{s_X\eta}{1-s_M}\right)\left(\frac{IM_0^M - IM_0^C\left(\frac{s_M}{s_C}\right)}{IM_0^C\left(\frac{s_X\eta}{s_C}\right)+EX_0\eta}\right)\right](1-\alpha)\bar{\chi} - \left(\frac{s_X\eta}{1-s_M}\right)\left(\frac{IM_0^M + IM_0^C\left(\frac{1-s_M}{s_C}\right)-TD_0}{IM_0^C\left(\frac{s_X\eta}{s_C}\right)+EX_0\eta}\right)\right]$.

Via Step Three, the dynamic IS Curve follows:

$$\hat{y}_t - \hat{y}_t^n = E_t\left[\hat{y}_{t+1} - \hat{y}_{t+1}^n\right] - \frac{1}{\bar{\theta}\rho}\left(\hat{\tilde{r}}_t - \hat{\tilde{r}}_t^n\right). \tag{107}$$

As in the Phillips Curve, the immediate effect of relaxing the complete markets assumption is to change the slope parameter. The deeper impact of this change in the model is hidden from view, embedded in $\hat{\tilde{r}}_t^n$.

To derive the real natural rate, we refer back to Equation 81. To solve for consumption growth in the flexible price equilibrium, we first use the output market clearing and trade deficit equation to eliminate the real exchange rate and link output and consumption. From output market clearing, the real exchange rate is given by:

$$\hat{q}_t^n = \left(\frac{1-s_M}{s_X\eta}\right)\hat{y}_t^n - \left(\frac{1-s_M}{s_X(\eta-1)}\right)\hat{\lambda}_{Ht}^C - \left(\frac{s_C}{s_X\eta}\right)\hat{c}_t^n - \frac{1}{\eta}\hat{c}_t^* - \left(\frac{s_M}{s_X\eta}\right)\hat{\lambda}_{Ht}^M. \tag{108}$$

Then substitute this into the trade balance condition and rearrange to get:

$$\hat{y}_t^n = \Upsilon_C^y\hat{\lambda}_{Ht}^C + \hat{c}_t^n + \Upsilon_M^y\hat{\lambda}_{Ht}^M + \Upsilon_{tdy}^y\widehat{tdy}_t, \tag{109}$$

where the coefficients are given by:

$$\Upsilon_C^y = \left[IM_0^C + EX_0\left(\frac{s_C}{s_X}\right)\right]^{-1}\left[-IM_0^C\left(\frac{\eta\lambda_{H0}^C - 1}{(\eta-1)(1-\lambda_{H0}^C)}\right) + EX_0\left(\frac{\eta}{\eta-1}\right)\left(\frac{s_C}{s_X}\right)\right]$$
$$\Upsilon_M^y = \left[IM_0^C + EX_0\left(\frac{s_C}{s_X}\right)\right]^{-1}EX_0\left(\frac{s_M}{s_X}\right) > 0$$
$$\Upsilon_{tdy}^y = -\left[IM_0^C + EX_0\left(\frac{s_C}{s_X}\right)\right]^{-1}TD_0 < 0 \quad \text{if } TD_0 > 0.$$

Now we use the production function, labor supply, and real marginal cost equations to link output and consumption. The real marginal cost equation (recognizing that $\widehat{rmc}_t = 0$ in the flexible price equilibrium) and the labor supply condition yield:

$$\hat{l}_t^n = -\frac{\rho}{\psi}\hat{c}_t^n - \frac{\alpha}{\psi(\eta-1)(1-\alpha)}\hat{\lambda}_{Ht}^M - \frac{1}{\psi(\eta-1)}\hat{\lambda}_{Ht}^C + \frac{1}{\psi(1-\alpha)}\hat{z}_t. \tag{110}$$

Then we plug this in to the production function to get:

$$\hat{y}_t^n = -\frac{\rho}{\psi}\hat{c}_t^n + \left(1+\frac{1}{\psi}\right)\left(\frac{1}{1-\alpha}\right)\hat{z}_t - \frac{\alpha}{(\eta-1)(1-\alpha)}\left(1+\frac{1}{\psi}\right)\hat{\lambda}_{Ht}^M - \frac{1}{\psi(\eta-1)}\hat{\lambda}_{Ht}^C. \tag{111}$$

We then combine Equations 109 and 111 to solve for consumption:

$$\hat{c}_t^n = \Upsilon_z^c \hat{z}_t + \Upsilon_M^c \hat{\lambda}_{Ht}^M + \Upsilon_C^c \hat{\lambda}_{Ht}^C + \Upsilon_{tdy}^c \widehat{tdy}_t, \tag{112}$$

where the coefficients are given by:

$$\begin{aligned}
\Upsilon_z^c &= \left(\frac{1+\psi}{\psi+\rho}\right)\left(\frac{1}{1-\alpha}\right) > 0 \\
\Upsilon_M^c &= -\frac{\psi}{\psi+\rho}\left[\frac{\alpha}{(\eta-1)(1-\alpha)}\left(\frac{1+\psi}{\psi}\right) + \Upsilon_M^y\right] < 0 \\
\Upsilon_C^c &= -\frac{\psi}{\psi+\rho}\left[\frac{1}{\psi(\eta-1)} + \Upsilon_C^y\right] \\
\Upsilon_\varepsilon^c &= -\frac{\psi}{\psi+\rho}\Upsilon_\varepsilon^y > 0.
\end{aligned}$$

The final step is then to insert this solution for consumption into the Euler Equation, and solve for the natural real interest rate:

$$\hat{\tilde{r}}_t^n = \tilde{\Upsilon}_z E_t \Delta \ln z_{t+1} + \tilde{\Upsilon}_M E_t \Delta \ln \lambda_{Ht+1}^M + \tilde{\Upsilon}_C E_t \Delta \ln \lambda_{Ht+1}^C + \tilde{\Upsilon}_{tdy} E_t \Delta \ln tdy_{t+1}, \tag{113}$$

with $\tilde{\Upsilon}_z = \rho\Upsilon_z^c > 0$, $\tilde{\Upsilon}_M = \rho\Upsilon_M^c < 0$, $\tilde{\Upsilon}_C = \rho\Upsilon_C^c$, and $\tilde{\Upsilon}_{tdy}^c = \rho\Upsilon_{tdy}^c > 0$. And $\tilde{\Upsilon}_C < 0$ if $IM_0^C \frac{(\rho+1)(\eta\lambda_{H0}^C-1)}{(\eta-1)(1-\lambda_{H0}^C)} > \frac{X_0}{(\eta-1)}\frac{s_C}{s_X}(\rho\eta+1)$. The new result here is that an expected increase in the trade deficit ($E_t\Delta \ln tdy_{t+1}$) raises the natural rate of interest.

E Model with Variable Markups

Drawing on Section 3.2, we briefly describe new equilibrium conditions for the model with Kimball demand and dollar currency pricing. Consumers choose consumption of individual home and foreign varieties to minimize expenditure with the consumption aggregator given by Equation 38. In a symmetric firm equilibrium, this yields the following equilibrium conditions:

$$C_{Ht} = \nu \Psi \left(\frac{D_{Ct} P_{Ht}}{P_{Ct}} \right) C_t \tag{114}$$

$$C_{Ft} = (1-\nu) \Psi \left(\frac{D_{Ct} \tau_{Ct} P_{Ft}}{P_{Ct}} \right) C_t \tag{115}$$

$$\nu \Upsilon \left(\frac{C_{Ht}}{\nu C_t} \right) + (1-\nu) \Upsilon \left(\frac{C_{Ft}}{(1-\nu) C_t} \right) = 1 \tag{116}$$

$$P_{Ct} C_t = P_{Ht} C_{Ht} + \tau_{Ct} P_{Ft} C_{Ft}, \tag{117}$$

where $\Psi(x) \equiv \Upsilon'^{-1}(x)$. These replace first order conditions for the consumption allocation and the consumer price index in the baseline model.

On the production side, Home producers choose home and foreign input use to minimize costs given the input aggregator in Equation 39, and they set prices for sales to domestic buyers and export buyers separately. The new equilibrium conditions are:

$$M_{Ht} = \xi \Psi \left(\frac{D_{Mt} P_{Ht}}{P_{Mt}} \right) M_t \tag{118}$$

$$M_{Ft} = (1-\xi) \Psi \left(\frac{D_{Mt} \tau_{Mt} P_{Ft}}{P_{Mt}} \right) M_t \tag{119}$$

$$\xi \Upsilon \left(\frac{M_{Ht}}{\xi M_t} \right) + (1-\xi) \Upsilon \left(\frac{M_{Ft}}{(1-\xi) M_t} \right) = 1 \tag{120}$$

$$P_{Mt} M_t = P_{Ht} M_{Ht} + \tau_{Mt} P_{Ft} M_{Ft}. \tag{121}$$

We assume that exporters face a constant elasticity demand curve in the foreign market, such that $X_t = \left(\frac{P_{Xt}}{E_t P_t^*} \right)^{-\epsilon_X} C_t^*$, similar to the baseline model. The (symmetric) firm's optimal prices then satisfy the following dynamic equation in the domestic market:

$$\begin{aligned} 0 = 1 - \epsilon_{Ht} \left(1 - \frac{MC_t}{P_{Ht}} \right) - \phi \left(\frac{P_{Ht}}{P_{Ht-1}} - 1 \right) \left(\frac{P_{Ht}}{P_{Ht-1}} \right) \\ + \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} \phi \left(\frac{P_{Ht+1}}{P_{Ht}} - 1 \right) \frac{P_{H,t+1} Y_{Ht+1}}{P_{Ht} Y_{Ht}} \frac{P_{Ht+1}}{P_{Ht}} \right], \end{aligned} \tag{122}$$

where $\epsilon_{Ht} = - \left[\Xi_\Psi \left(\frac{D_{Ct} P_{Ht}}{P_{Ct}} \right) \frac{C_{Ht}}{Y_{Ht}} + \Xi_\Psi \left(\frac{D_{Mt} P_{Ht}}{P_{Mt}} \right) \frac{M_{Ht}}{Y_{Ht}} \right]$ is the elasticity of demand at Home, with $\Xi_\Psi(x) \equiv \frac{\Psi'(x)}{\Psi(x)} x$ and $Y_{Ht} = C_{Ht} + M_{Ht}$. With the Klenow-Willis Υ-function, the elasticity of demand for Home goods by Home buyers is: $\epsilon_{Ht} = \frac{C_{Ht}}{Y_{Ht}} \epsilon_{Ht}^C + \frac{M_{Ht}}{Y_{Ht}} \epsilon_{Ht}^M$, with $\epsilon_{Ht}^C = \sigma \left(1 + \varepsilon \ln \frac{\sigma-1}{\sigma} - \varepsilon \ln \frac{D_{Ct} P_{Ht}}{P_{Ct}} \right)^{-1}$ and $\epsilon_{Ht}^M = \sigma \left(1 + \varepsilon \ln \frac{\sigma-1}{\sigma} - \varepsilon \ln \frac{D_{Mt} P_{Ht}}{P_{Mt}} \right)^{-1}$.

The firm's optimal prices in the export market are given by:

$$
\begin{aligned}
0 = 1 - \epsilon_X \left(1 - \frac{MC_t}{P_{Xt}}\right) - \phi \left(\frac{P_{Xt}}{P_{Xt-1}} - 1\right)\left(\frac{P_{Xt}}{P_{Xt-1}}\right) \\
+ \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{P_{Ct}}{P_{C,t+1}} \phi \left(\frac{P_{Xt+1}}{P_{Xt}} - 1\right) \frac{P_{Xt+1}X_{t+1}}{P_{Xt}X_t} \frac{P_{Xt+1}}{P_{Xt}}\right],
\end{aligned} \tag{123}
$$

where ϵ_X is the constant elasticity of export demand.

On the import side, we assume foreign producers set import prices in dollars (exclusive of trade costs), subject to adjustment costs. Their optimal pricing rule is analogous to the Home firms:

$$
\begin{aligned}
0 = 1 - \epsilon_{Ft} \left(1 - \frac{E_t MC_t^*}{P_{Ft}}\right) - \phi \left(\frac{P_{Ft}}{P_{Ft-1}} - 1\right)\left(\frac{P_{Ft}}{P_{Ft-1}}\right) \\
+ \beta E_t \left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho} \frac{E_t P_{Ct}^*}{E_{t+1} P_{Ct+1}^*} \phi \left(\frac{P_{Ft+1}}{P_{Ft}} - 1\right) \frac{P_{F,t+1}Y_{Ft+1}}{P_{Ft}Y_{Ft}} \left(\frac{P_{Ft+1}}{P_{Ft}}\right)\right],
\end{aligned} \tag{124}
$$

where $\epsilon_{Ft} = -\left[\Xi_\Psi \left(\frac{D_{Ct}\tau_{Ct}P_{Ft}}{P_{Ct}}\right) \frac{C_{Ft}}{Y_{Ft}} + \Xi_\Psi \left(\frac{D_{Mt}\tau_{Mt}P_{Ft}}{P_{Mt}}\right) \frac{M_{Ft}}{Y_{Ft}}\right]$ is the elasticity of import demand and $Y_{Ft} = \tau_{Ct}C_{Ft} + \tau_{Mt}M_{Ft}$.

Collecting and log-linearizing the model equilibrium conditions yields the system in Table 5. To reduce and simplify this system, we make the following observations.

First, we calibrate the model so that preference parameters ν and ζ match the domestic shares of final and input expenditure. With an appropriate choice of units, we have $P_{H0}/P_{C0} = \tau_{C0}P_{F0}/P_{C0} = 1$ and $P_{H0}/P_{M0} = \tau_{M0}P_{F0}/P_{M0} = 1$, so $C_{H0} = \nu C_0$, $C_{F0} = (1-\nu)C_0$, $M_{H0} = \zeta M_0$, and $M_{F0} = (1-\zeta)M_0$.

Second, with this result in hand, it is possible to show that $\hat{d}_{Ct} = 0$ and $\hat{d}_{Mt} = 0$ in any equilibrium. Working first with consumption, the final goods aggregator implies that aggregate consumption satisfies $\hat{c}_t = \frac{C_{H0}}{C_0}\hat{c}_{Ht} + \frac{C_{F0}}{C_0}\hat{c}_{Ft}$. Given the nominal spending identity $P_{Ct}C_t = P_{Ht}C_{Ht} + \tau_{Ct}P_{Ft}C_{Ft}$, then the price index can be expressed as $\hat{p}_{Ct} = \frac{C_{H0}}{C_0}\hat{p}_{Ht} + \frac{C_{F0}}{C_0}\left(\hat{\tau}_{Ct} + \hat{p}_{Ft}\right)$, where $\frac{P_{H0}}{P_{C0}} = 1$ and $\frac{\tau_{C0}P_{F0}}{P_{C0}} = 1$. Then, plugging the first order conditions into the consumption aggregator and simplifying yields:

$$
0 = \frac{C_{H0}}{C_0}\left[\left(\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct}\right)\right] + \frac{C_{F0}}{C_0}\left[\left(\hat{d}_{Ct} + \hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct}\right)\right] = \hat{d}_{Ct},
$$

where the second equality uses the prior result for $\hat{p}_{Ct}$. An identical procedure applied to inputs then returns $\hat{d}_{Mt} = 0$ as well.

Third, we can draw on the arguments in the text and these first two results to write relative prices as functions of changes in domestic sourcing shares:

$$
\hat{p}_{Ht} - \hat{p}_{Ct} = -\frac{1}{\sigma - 1}\hat{\lambda}_{Ht}^C \tag{125}
$$

$$
\hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct} = -\frac{1}{\sigma - 1}\hat{\lambda}_{Ft}^C = -\frac{1}{\sigma - 1}\frac{\lambda_{F0}}{\lambda_{H0}}\hat{\lambda}_{Ht}^C \tag{126}
$$

$$
\hat{p}_{Ht} - \hat{p}_{Mt} = -\frac{1}{\sigma - 1}\hat{\lambda}_{Ht}^M \tag{127}
$$

$$
\hat{\tau}_{Mt} + \hat{p}_{Ft} - \hat{p}_{Mt} = -\frac{1}{\sigma - 1}\hat{\lambda}_{Ft}^M = -\frac{1}{\sigma - 1}\frac{\lambda_{F0}}{\lambda_{H0}}\hat{\lambda}_{Ht}^M, \tag{128}
$$

where we have used $\lambda_{Ht}^C + \lambda_{Ft}^C = 1$ and $\lambda_{Ht}^M + \lambda_{Ft}^M = 1$.

Fourth, given this rewriting, we can solve a subset of the equilibrium system to determine inflation. In particular, we can drop equations that pertain to demand for foreign final goods and inputs, and we can drop the dynamic pricing equation for imports and associated definitions of the elasticity of demand for imports.

Together, these four sets of results imply we can collapse down the equilibrium into the system presented in Table 6. In the table, $\widehat{rp}_{Xt} \equiv \hat{p}_{Xt} - \hat{p}_{Ht}$, and other variables match definitions in the baseline model.

A few additional words are helpful to interpret how we write the elasticity faced by domestic firms ($\hat{\epsilon}_{Ht}$) in this table, and this discussion helps one interpret Equations 43-45 in the main text as well. In the steady state, $\epsilon^C_{H0} = \epsilon^M_{H0} = \sigma$. A sketch proof of this statement is as follows. Using the Klenow-Willis functional form, the first order condition for consumption is $\frac{C_{H0}}{\nu C_0} = \left(1 + \varepsilon \ln\left(\frac{\sigma-1}{\sigma}\right) - \varepsilon \ln\left(\frac{D_{C0}P_{H0}}{P_{C0}}\right)\right)^{\sigma/\varepsilon}$, where we have evaluated it in the steady state. Since $\frac{P_{H0}}{P_{C0}} = 1$ and $\frac{C_{H0}}{\nu C_0} = 1$ in the steady state, then $D_{C0} = \frac{\sigma-1}{\sigma}$. A parallel argument for inputs implies that $D_{M0} = \frac{\sigma-1}{\sigma}$ too. Then $\epsilon^C_{H0} = \sigma\left(1 + \varepsilon \ln \frac{\sigma-1}{\sigma} - \varepsilon \ln \frac{D_{C0}P_{H0}}{P_{C0}}\right)^{-1} = \sigma$ and $\epsilon^M_{H0} = \sigma\left(1 + \varepsilon \ln \frac{\sigma-1}{\sigma} - \varepsilon \ln \frac{D_{M0}P_{H0}}{P_{M0}}\right)^{-1} = \sigma$. Given this result, $\frac{C_{H0}}{Y_{H0}} \frac{\epsilon^C_{H0}}{\epsilon_{H0}} \left(\hat{c}_{Ht} - \hat{y}_{Ht}\right) + \frac{M_{H0}}{Y_{H0}} \frac{\epsilon^M_{H0}}{\epsilon_{H0}} \left(\hat{m}_{Ht} - \hat{y}_{Ht}\right) = 0$, since $\frac{C_{H0}}{Y_{H0}}\hat{c}_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{m}_{Ht} = \hat{y}_{Ht}$. So, $\hat{\epsilon}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{\epsilon}^C_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{\epsilon}^M_{Ht}$, as in Equation 43. Further, the same results lead $\hat{\epsilon}^C_{Ht}$ and $\hat{\epsilon}^M_{Ht}$to simplify as well, where only the parameter ε governs how relative prices influence deviations in elasticities from steady state.

Table 5: Log-Linearized Variable Markups Model Equilibrium

Consumption-Leisure	$-\frac{\rho}{\psi}\hat{c}_t + \frac{1}{\psi}\left[\hat{w}_t - \hat{p}_{Ht}\right] + \frac{1}{\psi}\left(\hat{p}_{Ht} - \hat{p}_{Ct}\right) = \hat{l}_t$
Consumption Allocation	$\hat{c}_{Ht} = -\sigma\left(\frac{C_{H0}}{\nu C_0}\right)^{-\varepsilon/\sigma}\left(\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct}\right) + \hat{c}_t$
	$\hat{c}_{Ft} = -\sigma\left(\frac{C_{F0}}{(1-\nu)C_0}\right)^{-\varepsilon/\sigma}\left(\hat{d}_{Ct} + \hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct}\right) + \hat{c}_t$
	$0 = \exp\left(-\frac{1}{\varepsilon}\left(\frac{C_{H0}}{\nu C_0}\right)^{\varepsilon/\sigma}\right)\frac{C_{H0}}{C_0}\left[\hat{c}_{Ht} - \hat{c}_t\right] + \exp\left(-\frac{1}{\varepsilon}\left(\frac{C_{F0}}{(1-\nu)C_0}\right)^{\varepsilon/\sigma}\right)\frac{C_{F0}}{C_0}\left[\hat{c}_{Ft} - \hat{c}_t\right]$
Euler Equation	$\hat{c}_t = E_t\left(\hat{c}_{t+1} - (r_t - \pi_{Ct+1})/\rho\right)$
Input Choices	$\left[\hat{w}_t - \hat{p}_{Ht}\right] + \hat{l}_t = \left[\hat{mc}_t - \hat{p}_{Ht}\right] + \hat{y}_t$
	$\left[\hat{p}_{Mt} - \hat{p}_{Ht}\right] + \hat{m}_t = \left[\hat{mc}_t - \hat{p}_{Ht}\right] + \hat{y}_t$
	$\hat{m}_{Ht} = -\sigma\left(\frac{M_{H0}}{\zeta M_0}\right)^{-\varepsilon/\sigma}\left(\hat{d}_{Mt} + \hat{p}_{Ht} - \hat{p}_{Mt}\right) + \hat{m}_t$
	$\hat{m}_{Ft} = -\sigma\left(\frac{M_{F0}}{(1-\zeta)M_0}\right)^{-\varepsilon/\sigma}\left(\hat{d}_{Mt} + \hat{\tau}_{Mt} + \hat{p}_{Ht} - \hat{p}_{Mt}\right) + \hat{m}_t$
	$0 = \exp\left(-\frac{1}{\varepsilon}\left(\frac{M_{H0}}{\zeta M_0}\right)^{\varepsilon/\sigma}\right)\frac{M_{H0}}{M_0}\left[\hat{m}_{Ht} - \hat{m}_t\right] + \exp\left(-\frac{1}{\varepsilon}\left(\frac{M_{F0}}{(1-\zeta)M_0}\right)^{\varepsilon/\sigma}\right)\frac{M_{F0}}{M_0}\left[\hat{m}_{Ft} - \hat{m}_t\right]$
Marginal Cost	$\hat{mc}_t - \hat{p}_{Ht} = -\hat{z}_t + (1-\alpha)\left(\hat{w}_t - \hat{p}_{Ht}\right) + \alpha\left(\hat{p}_{Mt} - \hat{p}_{Ht}\right)$
Domestic Price Setting	$\pi_{Ht} = \frac{-\hat{\epsilon}_{Ht}}{\phi} + \frac{(\epsilon_{H0}-1)}{\phi}\left(\hat{mc}_t - \hat{p}_{Ht}\right) + \beta E_t\left(\pi_{Ht+1}\right)$
Import Price Setting	$\pi_{Ft} = \frac{-\hat{\epsilon}_{Ft}}{\phi} + \frac{(\epsilon_{F0}-1)}{\phi}\left[\hat{q}_t - \left(\hat{p}_{Ft} - \hat{p}_{Ct}\right) + \left(\hat{mc}^*_t - \hat{p}^*_{Ct}\right)\right] + \beta E_t\left(\pi_{Ft+1}\right)$
Export Price Setting	$\pi_{Xt} = \frac{(\epsilon_X-1)}{\phi}\left[\left(\hat{mc}_t - \hat{p}_{Ht}\right) - \left(\hat{p}_{Xt} - \hat{p}_{Ht}\right)\right] + \beta E_t\left(\pi_{Xt+1}\right)$
Auxiliary Inflation Definitions	$\pi_{Ht} = \left[\left(\hat{p}_{Ht} - \hat{p}_{Ct}\right) - \left(\hat{p}_{Ht-1} - \hat{p}_{Ct-1}\right)\right] + \pi_{Ct}$
	$\pi_{Xt} = \left[\left(\hat{p}_{Xt} - \hat{p}_{Ht}\right) - \left(\hat{p}_{Xt-1} - \hat{p}_{Ht-1}\right)\right] + \pi_{Ht}$
	$\pi_{Ft} = \left[\left(\hat{p}_{Ft} - \hat{p}_{Ct}\right) - \left(\hat{p}_{Ft-1} - \hat{p}_{Ct-1}\right)\right] + \pi_{Ct}$
Elasticities	$\hat{\epsilon}_{Ht} = \left[\frac{C_{H0}}{Y_{H0}}\frac{\epsilon^C_{H0}}{\epsilon_{H0}}\hat{\epsilon}^C_{Ht} + \frac{M_{H0}}{Y_{H0}}\frac{\epsilon^M_{H0}}{\epsilon_{H0}}\hat{\epsilon}^M_{Ht}\right] + \frac{C_{H0}}{Y_{H0}}\frac{\epsilon^C_{H0}}{\epsilon_{H0}}\left(\hat{c}_{Ht} - \hat{y}_{Ht}\right) + \frac{M_{H0}}{Y_{H0}}\frac{\epsilon^M_{H0}}{\epsilon_{H0}}\left(\hat{m}_{Ht} - \hat{y}_{Ht}\right)$
	with $\hat{\epsilon}^C_{Ht} = -\frac{\varepsilon}{\sigma}\epsilon^C_{H0}\left(\hat{d}_{Ct} + \hat{p}_{Ht} - \hat{p}_{Ct}\right)$
	and $\hat{\epsilon}^M_{Ht} = -\frac{\varepsilon}{\sigma}\epsilon^M_{H0}\left(\hat{d}_{Mt} + \hat{p}_{Ht} - \hat{p}_{Mt}\right)$
	$\hat{\epsilon}_{Ft} = \left[\frac{C_{F0}}{Y_{F0}}\frac{\epsilon^C_{F0}}{\epsilon_{F0}}\hat{\epsilon}^C_{Ft} + \frac{M_{F0}}{Y_{F0}}\frac{\epsilon^M_{F0}}{\epsilon_{F0}}\hat{\epsilon}^M_{Ft}\right] + \frac{C_{F0}}{Y_{F0}}\frac{\epsilon^C_{F0}}{\epsilon_{F0}}\left(\hat{c}_{Ft} - \hat{y}_{Ft}\right) + \frac{M_{F0}}{Y_{F0}}\frac{\epsilon^M_{F0}}{\epsilon_{F0}}\left(\hat{m}_{Ft} - \hat{y}_{Ft}\right)$
	with $\hat{\epsilon}^C_{Ft} = -\frac{\varepsilon}{\sigma}\epsilon^C_{H0}\left(\hat{d}_{Ct} + \hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct}\right)$
	and $\hat{\epsilon}^M_{Ft} = -\frac{\varepsilon}{\sigma}\epsilon^M_{H0}\left(\hat{d}_{Mt} + \hat{\tau}_{Mt} + \hat{p}_{Ft} - \hat{p}_{Mt}\right)$
Price Indexes	$\hat{c}_t = \left(\frac{P_{H0}C_{H0}}{P_{C0}C_0}\right)\left(\hat{p}_{Ht} - \hat{p}_{Ct}\right) + \left(\frac{\tau_{C0}P_{F0}C_{F0}}{P_{C0}C_0}\right)\left(\hat{\tau}_{Ct} + \hat{p}_{Ft} - \hat{p}_{Ct}\right) + \left(\frac{P_{H0}C_{H0}}{P_{C0}C_0}\right)\hat{c}_{Ht} + \left(\frac{\tau_{C0}P_{F0}C_0}{P_{C0}C_0}\right)\hat{c}_{Ft}$
	$\hat{m}_t = \left(\frac{P_{H0}M_{H0}}{P_{M0}M_0}\right)\left(\hat{p}_{Ht} - \hat{p}_{Mt}\right) + \left(\frac{\tau_{M0}P_{F0}M_{F0}}{P_{M0}M_0}\right)\left(\hat{\tau}_{Mt} + \hat{p}_{Ft} - \hat{p}_{Mt}\right) + \left(\frac{P_{H0}M_{H0}}{P_{M0}M_0}\right)\hat{m}_{Ht} + \left(\frac{\tau_{M0}P_{F0}M_{F0}}{P_{M0}M_0}\right)\hat{m}_{Ft}$
Market Clearing	$\hat{y}_t = \frac{Y_{H0}}{Y_0}\hat{y}_{Ht} + \frac{X_0}{Y_0}\hat{x}_t$
	$\hat{y}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{c}_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{m}_{Ht}$
	$\hat{y}_{Ft} = \frac{C_{F0}}{Y_{F0}}\hat{c}_{Ht} + \frac{M_{F0}}{Y_{F0}}\hat{m}_{Ft}$
Exports	$\hat{x}_t = -\epsilon_X\left(\left(\hat{p}_{Xt} - \hat{p}_{Ht}\right) + \left(\hat{p}_{Ht} - \hat{p}_{Ct}\right) - \hat{q}_t\right) + \hat{c}^*_t$
Complete Asset Markets	$\hat{c}_t = \hat{c}^*_t + \frac{1}{\rho}\hat{q}_t$
Monetary Policy Rule	$\hat{r}_t = \omega\pi_{Ct}$

Table 6: Simplified VM Model with Domestic Sourcing Shocks

Consumption-Leisure	$-\frac{\rho}{\psi}\hat{c}_t + \frac{1}{\psi}\widehat{rw}_t - \frac{1}{\psi(\sigma-1)}\hat{\lambda}^C_{Ht} = \hat{l}_t$
Consumption Allocation	$\hat{c}_{Ht} = \frac{\sigma}{\sigma-1}\hat{\lambda}^C_{Ht} + \hat{c}_t$
Euler Equation	$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\rho}\left(\hat{r}_t - E_t\pi_{Ct+1}\right)$
Input Choices	$\hat{l}_t = \widehat{rmc}_t + \hat{y}_t - \widehat{rw}_t$
	$\hat{m}_t = \widehat{rmc}_t + \hat{y}_t - \frac{1}{\sigma-1}\hat{\lambda}^M_{Ht}$
	$\hat{m}_{Ht} = \frac{\sigma}{\sigma-1}\hat{\lambda}^M_{Ht} + \hat{m}_t$
Marginal Cost	$\widehat{rmc}_t = (1-\alpha)\widehat{rw}_t + \frac{\alpha}{\sigma-1}\hat{\lambda}^M_{Ht} - \hat{z}_t$
Domestic Price Inflation	$\pi_{Ht} = -\frac{1}{\phi}\hat{\epsilon}_{Ht} + \frac{(\epsilon_{H0}-1)}{\phi}\widehat{rmc}_t + \beta E_t\left(\pi_{Ht+1}\right)$
Export Price Inflation	$\pi_{Xt} = \left(\frac{\epsilon_X - 1}{\phi}\right)\left(\widehat{rmc}_t - \widehat{rp}_{Xt}\right) + \beta E_t\left(\pi_{Xt+1}\right)$
	with $\pi_{Xt} = \left[\widehat{rp}_{Xt} - \widehat{rp}_{Xt-1}\right] + \pi_{Ht}$
Consumer Price Inflation	$\pi_{Ct} = \pi_{Ht} + \frac{1}{\sigma-1}\left(\hat{\lambda}^C_{Ht} - \hat{\lambda}^C_{Ht-1}\right)$
Elasticities	$\hat{\epsilon}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{\epsilon}^C_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{\epsilon}^M_{Ht}$
	with $\hat{\epsilon}^C_{Ht} = -\left(\frac{\varepsilon}{\sigma-1}\right)\hat{\lambda}^C_{Ht}$
	and $\hat{\epsilon}^M_{Ht} = -\left(\frac{\varepsilon}{\sigma-1}\right)\hat{\lambda}^M_{Ht}$
Market Clearing	$\hat{y}_t = \frac{Y_{H0}}{Y_0}\hat{y}_{Ht} + \frac{X_0}{Y_0}\hat{x}_t$
	$\hat{y}_{Ht} = \frac{C_{H0}}{Y_{H0}}\hat{c}_{Ht} + \frac{M_{H0}}{Y_{H0}}\hat{m}_{Ht}$
	$\hat{x}_t = -\epsilon_X\widehat{rp}_{Xt} + \frac{\epsilon_X}{\sigma-1}\hat{\lambda}^C_{Ht} + \epsilon_X\hat{q}_t + \hat{c}^*_t$
	$\hat{c}_t = \hat{c}^*_t + \frac{1}{\rho}\hat{q}_t$
Monetary Policy Rule	$\hat{r}_t = \omega\pi_{Ct}$

F Multisector Model

The structure of the multisector model follows the one sector model closely, so we emphasize modifications in our discussion here.

Consumers Consumer preferences are given by Equation 18, with aggregate consumption defined as in Equations 47-48. The consumers intertemporal budget constraint is adjusted for the multisector structure:

$$\sum_s P_{Ht}(s)C_{Ht}(s) + \sum_s P_{Ft}(s)\tau_{Ct}(s)C_{Ft}(s) + E_t\left[Q_{t,t+1}D_{t+1}\right] \leq D_t + W_t L_t, \tag{129}$$

where the prices of the composite goods are $\{P_{Ht}(s), P_{Ft}(s)\}$. The parameter $\tau_{Ct}(s)$ is an iceberg trade cost, paid on imports.

Given $\{P_{Ht}(s), P_{Ft}(s), Q_{t,t+1}, W_t\}$ and initial asset holdings, the consumer's problem is to choose $\{C_t, C_{Ht}(s), C_{Ft}(s), L_t, D_{t+1}\}$ to maximize 18 given 47-48 and subject to 129 and the standard transversality condition.

Producers Similar to the baseline model, there is a unit continuum of varieties in each sector, which are produced under monopolistic competition. To simplify the notation, we assume that these varieties are aggregated into composite goods, which are then consumed at home and exported.[39]

Varieties are aggregated by competitive intermediary firms into sector-level composites with the technology:

$$Y_t(s) = \left(\int_0^1 Y_t(s,i)^{(\epsilon(s)-1)/\epsilon_t(s)} dj\right)^{\epsilon(s)/(\epsilon(s)-1)}, \tag{130}$$

where $Y_t(s,i)$ is the quantity of variety i used to produce the composite Home good and $\epsilon(s)$ is the elasticity for sector s. Given prices $\{P_{Ht}(s,i)\}$ for individual varieties, cost minimization by the intermediaries yields these first order conditions and price indexes: $Y_t(s,i) = \left(\frac{P_t(s,i)}{P_{Ht}(s)}\right)^{-\epsilon(s)} Y_t(s)$ and $P_{Ht}(s) = \left[\int_0^1 P_{Ht}(s,i)^{1-\epsilon(s)} di\right]^{1/(1-\epsilon(s))}$.

The production function for individual varieties is given by Equations 50-52. Producers of differentiated output set the prices of their goods taking as given the demand and select the input mix to satisfy the implied demand. The firm chooses a sequence for $P_{Ht}(s,i)$ to maximize:

$$E\sum_{t=0}^{\infty} \beta^t \frac{C_t^{-\rho}}{C_0^{-\rho}} \frac{1}{P_{Ct}} \left[P_{Ht}(s,i)Y_t(s,i) - MC_t(s,i)Y_t(s,i) - \frac{\phi}{2}\left(\frac{P_{Ht}(s,i)}{P_{H,t-1}(s,i)} - 1\right)^2 P_{Ht}(s)Y_t(s)\right], \tag{131}$$

where $MC_t(s,i)$ is the constant marginal costs of the firm (defined below). Further, firm i in sector s chooses $\{L_t(s,i), M_t(s,i), M_{Ht}(s,i), M_{Ft}(s,i)\}$ to minimize the cost of producing a given amount of output $Y_t(s,i)$. Like consumers, the firm must pay iceberg trade costs to import inputs, given by $\tau_{Mt}(s)$, where s denotes the source sector of the goods.

[39] In contrast, in the baseline model we define preferences and technologies over varieties directly. Here we move aggregation of varieties into a separate production sector to lighten the notation.

Closing the Model Output market clearing for composite goods is:

$$Y_t(s) = C_{Ht}(s) + \sum_{s'} \int_0^1 M_{Ht}(s, s', j)dj + X_t(s) + \int_0^1 \frac{\phi}{2} \left(\frac{P_{Ht}(s,i)}{P_{H,t-1}(s,i)} - 1 \right)^2 Y_t(s)di \tag{132}$$

where $X_t(s)$ is sector-level exports. We again assume that demand for exports has a CES structure: $X_t(s) = \left(\frac{P_{Ht}(s)}{S_t P_t^*(s)} \right)^{-\eta(s)} C_t^*(s)$ with $P_t^*(s) C_t^*(s) = \gamma(s) P_t^* C_t^*$ where the export demand elasticity is the same as that between Home and Foreign goods for domestic agents.[40]

The labor market clearing condition is:$\sum_s \int_0^1 L_t(s,i) = L_t$. The international risk sharing condition applies, as in the baseline model. And we close the model via the same Taylor rule for domestic interest rates.

Equilibrium For reference, we collect equilibrium conditions in Table 7, without imposing a price normalization.

As in the baseline model, we work with the model equilibrium written in terms of domestic sourcing shares. $\Lambda_{Ct}(s) \equiv \frac{P_{Ht}(s) C_{Ht}(s)}{P_{Ct}(s) C_t(s)}$ and $\Lambda_{Mt}(s', s) = \frac{P_{Ht}(s') M_{Ht}(s',s)}{P_{Mt}(s',s) M_t(s',s)}$. Using first order conditions, we can related equilibrium prices to these shares as follows:

$$\frac{P_{Ht}(s)}{P_{Ct}(s)} = \left(\frac{\Lambda_{Ht}^C(s)}{\nu(s)} \right)^{1/(1-\eta(s))} \tag{133}$$

$$\frac{P_{Ht}(s')}{P_{Mt}(s',s)} = \left(\frac{\Lambda_{Ht}^M(s',s)}{\xi(s',s)} \right)^{1/(1-\eta(s'))}. \tag{134}$$

Thus, we can swap out for $\frac{P_{Ht}(s)}{P_{Ct}(s)}$ and $\frac{P_{Ht}(s')}{P_{Mt}(s',s)}$ throughout the equilibrium system.

We collect log-linearized equilibrium conditions in Table 8. In the table, we define relative prices as follows: $\widehat{rw}_t \equiv \hat{w}_t - \hat{p}_{Ct}$, $\widehat{rp}_{Ct}(s) \equiv \hat{p}_{Ct}(s) - \hat{p}_{Ct}$, $\widehat{rmc}_t(s) \equiv \widehat{mc}_t(s) - \hat{p}_{Ht}(s)$, $\widehat{rpm}_t(s) \equiv \hat{p}_{Mt}(s) - \hat{p}_{Ct}$, and $\widehat{rp}_t(s', s) \equiv \hat{p}_t(s', s) - \hat{p}_{Mt}(s)$. Given parameters, exogenous variables (foreign variables and domestic productivity), and domestic sourcing shares $\hat{\lambda}_{Ht}^C(s)$ and $\hat{\lambda}_{Ht}^M(s', s)$, an equilibrium is a path for prices $\{\widehat{rw}_t, \widehat{rp}_{Ct}(s), \widehat{rmc}_t(s), \widehat{rpm}_t(s), \widehat{rp}_t(s', s), \hat{r}_t, \hat{q}_t, \pi_{Ht}(s), \pi_t\}$ and quantities $\left\{\hat{c}_t, \hat{l}_t, \hat{c}_t(s), \hat{c}_{Ht}(s), \hat{l}_t(s), \hat{y}_t(s), \hat{m}_t(s), \hat{m}_t(s', s), \hat{m}_{Ht}(s', s), \hat{x}_t(s)\right\}$ that solve the dynamic system in Table 8.

To simulate the model, we set parameters already defined in Table 3 to the same values, and we set $\eta(s) = \epsilon(s) = 3$, except where noted in the text. We set parameters that govern the multisector input-

[40] As in the baseline model, P_t^* and C_t^* are aggregate foreign prices and consumption. The model now accomodates sector-level shocks, via foreign sector-level prices $P_t^*(s)$.

output structure to match data for 1996, which are given as follows:

$$\begin{bmatrix}\zeta(1)\\ \zeta(2)\end{bmatrix}=\begin{bmatrix}0.1854\\ 0.8146\end{bmatrix}$$

$$\begin{bmatrix}\alpha(1)\\ \alpha(2)\end{bmatrix}=\begin{bmatrix}0.7896\\ 0.4402\end{bmatrix}$$

$$\begin{bmatrix}\alpha(1,1) & \alpha(1,2)\\ \alpha(2,1) & \alpha(2,2)\end{bmatrix}=\begin{bmatrix}0.4868 & 0.0911\\ 0.3028 & 0.3491\end{bmatrix}$$

$$\begin{bmatrix}\nu(1)\\ \nu(2)\end{bmatrix}=\begin{bmatrix}0.7755\\ 0.9954\end{bmatrix}$$

$$\begin{bmatrix}\xi(1,1) & \xi(1,2)\\ \xi(2,1) & \xi(2,2)\end{bmatrix}=\begin{bmatrix}0.7221 & 0.7624\\ 0.8876 & 0.9815\end{bmatrix}.$$

Table 7: Equilibrium Conditions for the Multisector Model

Consumption-Leisure	$C_t^{-\rho}\frac{W_t}{P_{Ct}} = \mu L_t^{\psi}$
Consumption Allocation	$C_t(s) = \zeta(s)\left(\frac{P_{Ct}(s)}{P_{Ct}}\right)^{-\vartheta} C_t$
	$C_{Ht}(s) = \nu(s)\left(\frac{P_{Ht}(s)}{P_{Ct}(s)}\right)^{-\eta(s)} C_t(s)$
	$C_{Ft}(s) = (1-\nu(s))\left(\frac{\tau_{Ct}(s)P_{Ft}(s)}{P_{Ct}(s)}\right)^{-\eta(s)} C_t(s)$
Euler Equation	$1 = E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\frac{P_{Ct}}{P_{C,t+1}}(1+i_{t+1})\right]$
Input Choices	$W_tL_t(s) = (1-\alpha(s))MC_t(s)Y_t(s)$
	$P_{Mt}(s)M_t(s) = \alpha(s)MC_t(s)Y_t(s)$
	$M_t(s',s) = \frac{\alpha(s',s)}{\alpha(s)}\left(\frac{P_t(s',s)}{P_{Mt}(s)}\right)^{-\kappa} M_t(s)$
	$M_{Ht}(s',s) = \xi(s',s)\left(\frac{P_{Ht}(s')}{P_t(s',s)}\right)^{-\eta(s')} M_t(s',s)$
	$M_{Ft}(s',s) = (1-\xi(s',s))\left(\frac{\tau_{Mt}(s')P_{Ft}(s')}{P_t(s',s)}\right)^{-\eta_M(s')} M_t(s',s)$
Marginal Cost	$MC_t(s) = A\frac{W_t^{1-\alpha(s)}P_{Mt}(s)^{\alpha(s)}}{Z_t(s)}$ with $A(s)\equiv\alpha(s)^{-\alpha(s)}(1-\alpha(s))^{-(1-\alpha(s))}$
Input Prices	$P_{Mt}(s) = \left(\sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)P_t(s',s)^{1-\kappa}\right)^{1/(1-\kappa)}$
	$P_t(s',s) = \left[\xi(s',s)P_{Ht}(s')^{1-\eta(s')} + (1-\xi(s',s))\left(\tau_{Mt}(s')P_{Ft}(s')\right)^{1-\eta(s')}\right]^{1/(1-\eta(s'))}$
Domestic Pricing	$(1-\epsilon(s)) + \epsilon(s)\dfrac{MC_t(s)}{P_{Ht}(s)} - \phi\left(\dfrac{P_{Ht}(s)}{P_{H,t-1}(s)} - 1\right)\dfrac{P_{Ht}(s)}{P_{H,t-1}(s)}$
	$+E_t\left[\beta\dfrac{C_{t+1}^{-\rho}}{C_t^{-\rho}}\dfrac{P_{Ct}}{P_{C,t+1}}\phi\left(\dfrac{P_{H,t+1}(s)}{P_{H,t}(s)} - 1\right)\dfrac{P_{H,t+1}(s)Y_{t+1}(s)}{P_{Ht}(s)Y_t(s)}\dfrac{P_{H,t+1}(s)}{P_{Ht}(s)}\right] = 0$
Consumer Prices	$P_{Ct} = \left(\sum_s\zeta(s)P_{Ct}(s)^{1-\vartheta}\right)^{1/(1-\vartheta)}$
	$P_{Ct}(s) = \left(\nu(s)P_{Ht}(s)^{1-\eta(s)} + (1-\nu(s))\left(\tau_{Ct}(s)P_{Ft}(s)\right)\right)^{1/(1-\eta(s))}$
Market Clearing	$Y_t(s) = C_{Ht}(s) + \sum_{s'}M_{Ht}(s,s') + X_t(s) + \frac{\phi}{2}\left(\frac{P_{Ht}(s)}{P_{H,t-1}(s)} - 1\right)^2 Y_t(s)$
	$X_t(s) = \left(\frac{P_{Ht}(s)}{P_{Ct}Q_t}\right)^{-\eta(s)}\frac{\gamma(s)P_t^*C_t^*}{P_t^*(s)}$
	$\left(\frac{C_t}{C_t^*}\right)^{-\rho} Q_t = \Xi$
	$\sum_s L_t(s) = L_t$
Monetary Policy Rule	$1+i_t = (1+i_0)\left(\frac{P_{Ct}}{P_{C,t-1}}\right)^{\omega}$

Table 8: Log-Linearized Equilibrium Conditions for the Multisector Model

Consumption-Leisure	$-\frac{\rho}{\psi}\hat{c}_t + \frac{1}{\psi}\widehat{rw}_t = \hat{l}_t$
Consumption Allocation	$\hat{c}_t(s) = -\vartheta \widehat{rp}_{Ct}(s) + \hat{c}_t$
	$\hat{c}_{Ht}(s) = -\frac{\eta(s)}{\eta(s)-1}\hat{\lambda}^C_{Ht}(s) + \hat{c}_t(s)$
Euler Equation	$\hat{c}_t = E_t\left(\hat{c}_{t+1} - (r_t - \pi_{Ct+1})/\rho\right)$
	$\widehat{rw}_t + \hat{l}_t(s) = \widehat{rmc}_t(s) - \frac{1}{\eta(s)-1}\hat{\lambda}^C_{Ht}(s) + \widehat{rp}_{Ct}(s) + \hat{y}_t(s)$
Input Choices	$\widehat{rpm}_t(s) + \hat{m}_t(s) = \widehat{rmc}_t(s) - \frac{1}{\eta(s)-1}\hat{\lambda}^C_{Ht}(s) + \widehat{rp}_{Ct}(s) + \hat{y}_t(s)$
	$\hat{m}_t(s', s) = -\kappa \widehat{rp}_t(s', s) + \hat{m}_t(s)$
	$\hat{m}_{Ht}(s', s) = -\frac{\eta(s')}{\eta(s')-1}\hat{\lambda}^M_{Ht}(s', s) + \hat{m}_t(s', s)$
Real Marginal Cost	$\widehat{rmc}_t(s) - \dfrac{1}{\eta(s)-1}\hat{\lambda}^C_{Ht}(s) + \widehat{rp}_{Ct}(s) = (1-\alpha(s))\,\widehat{rw_t} + \alpha(s)\widehat{rpm}_t(s) - \hat{z}_t(s)$
Input Prices	$0 = \sum_{s'}\left(\frac{P_{M0}(s',s)M_0(s',s)}{P_{M0}(s)M_0(s)}\right)\widehat{rp}_t(s', s)$
	$\widehat{rp}_t(s', s) = \left(\frac{1}{\eta(s')-1}\right)\hat{\lambda}^M_{Ht}(s', s) - \frac{1}{\eta(s)-1}\hat{\lambda}^C_{Ht}(s) - \widehat{rp}_{Ct}(s)$
Domestic Pricing	$\pi_{Ht}(s) = \frac{(\epsilon(s)-1)}{\phi}\widehat{rmc}_t(s) + \beta E_t\left[\pi_{Ht+1}(s)\right]$
Consumer Prices	$0 = \sum_s \left(\frac{P_{C0}(s)C_0(s)}{P_{C0}C_0}\right)\widehat{rp}_{Ct}(s)$
	$\pi_{Ht}(s) = -\frac{1}{\eta(s)-1}\left(\hat{\lambda}^C_{Ht}(s) - \hat{\lambda}^C_{Ht-1}(s)\right) + \widehat{rp}_{Ct}(s) - \widehat{rp}_{Ct-1}(s) + \pi_t$
	$\hat{y}_t(s) = \frac{C_{H0}(s)}{Y_0(s)}\hat{c}_{Ht}(s) + \sum_{s'}\frac{M_{H0}(s,s^{'})}{Y_0(s)}\hat{m}_{Ht}(s, s^{'}) + \frac{X_0(s)}{Y_0(s)}\hat{x}_t(s)$
Market Clearing	$\hat{x}_t(s) = \frac{\eta(s)}{\eta(s)-1}\hat{\lambda}^C_{Ht}(s) - \eta(s)\widehat{rp}_{Ct}(s) + \eta(s)\hat{q}_t - (\hat{p}^*_t(s) - \hat{p}^*_{Ct}) + \hat{c}^*_t$
	$\hat{c}_t = \hat{c}^*_t + \frac{1}{\rho}\hat{q}_t$
	$\sum_s \frac{L_0(s)}{L_0}\hat{l}_t(s) = \hat{l}_t$
Monetary Policy Rule	$\hat{r}_t = \omega \pi_{Ct}$